Mathematik 2
2014-09-19
Musterlösungen

1) $$\begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 0-2 \\ 1-1 \\ 2-1 \end{pmatrix} + \mu \begin{pmatrix} 4-2 \\ 1-1 \\ 0-1 \end{pmatrix} \overset{?}{=} \begin{pmatrix} 5 \\ 4 \\ 3 \end{pmatrix}$$

$$\Leftrightarrow \underbrace{\lambda \begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}}_{\begin{pmatrix} * \\ 0 \\ * \end{pmatrix}} \overset{?}{=} \begin{pmatrix} 3 \\ 3 \\ 2 \end{pmatrix}$$

Also nein!

2) $$\begin{vmatrix} 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 3 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & a & 0 \end{vmatrix} = - \begin{vmatrix} 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & a & 0 \end{vmatrix} = - \begin{vmatrix} 2 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & a \end{vmatrix}$$

$= -6a$, also $a = -\frac{1}{6}$.

3) Allg. Lsg. von $y'' + y = 0$:

Ansatz: $y = e^{\lambda x}$

$\Rightarrow \lambda^2 + 1 = 0 \Rightarrow \lambda = \pm i$

Also $y(x) = Ae^{ix} + Be^{-ix}$

Eine spez. Lsg. von $y'' + y = x^2$:

Ansatz: $y(x) = Cx^2 + Dx + E$

$\Rightarrow 2C + Cx^2 + Dx + E = x^2$

$\Rightarrow C = 1,\ D = 0,\ E = -2$

$\Rightarrow$ allg. Lsg. von $y'' + y = x^2$:

$$y(x) = Ae^{ix} + Be^{-ix} + x^2 - 2$$

Wollen $y(0) = 5$ und $y'(0) = 0$.

$$\Leftrightarrow \begin{cases} Ae^{i0} + Be^{-i0} + 0^2 - 2 = 5 \\ A \cdot ie^{i0} + B(-i)e^{-i0} + 2 \cdot 0 = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} A + B - 2 = 5 \\ A - B = 0 \end{cases}$$

$$\Leftrightarrow A = B = \frac{7}{2}$$

Also $y(x) = \frac{7}{2}e^{ix} + \frac{7}{2}e^{-ix} + x^2 - 2$

$$\left(= 7\cos(x) + x^2 - 2 \right)$$

4) $y' = -y\sin(x) \Leftrightarrow \frac{y'}{y} = -\sin(x)$

$$\Leftrightarrow \text{„} \frac{dy}{y} = -\sin(x)\,dx \text{“}$$

$$\Rightarrow \underbrace{\int_3^{y_1} \frac{dy}{y}}_{\ln\left|\frac{y_1}{3}\right|} = \underbrace{-\int_1^{x_1} \sin(x)\,dx}_{-\cos(x_1) + \cos(1)}$$

überflüssig, weil $y > 0$

$$\Rightarrow y_1 = 3e^{\cos(x_1) - \cos(1)}$$

5)

$$f(x) = e^{1-\sqrt{x}}$$

$$f'(x) = e^{1-\sqrt{x}} \cdot \left(-\frac{1}{2\sqrt{x}}\right) \qquad \frac{1}{2}x^{-\frac{1}{2}}$$

$$f''(x) = e^{1-\sqrt{x}} \cdot \left(\left(-\frac{1}{2\sqrt{x}}\right)^2 - \frac{1}{2}\cdot\left(-\frac{1}{2}\right)x^{-\frac{3}{2}}\right)$$

$$f(1) = e^0 = 1$$

$$f'(1) = e^0 \cdot \left(-\frac{1}{2\sqrt{1}}\right) = -\frac{1}{2}$$

$$f''(1) = e^0 \cdot \left(\left(-\frac{1}{2}\right)^2 + \frac{1}{4}1^{-\frac{3}{2}}\right) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

$$f(0{,}99) \approx 1 - \frac{1}{2}\cdot(-0{,}01) + \frac{1}{2}\,\frac{(-0{,}01)^2}{2}$$

$$= 1 + 0{,}005 + 0{,}000025$$

$$= 1{,}005025$$

6)

An $\left(0 \middle| \frac{\pi}{4}\right)$:

$$\frac{\partial f}{\partial x} = \cos(x^2+y)2x \qquad = \cos\left(\frac{\pi}{4}\right)\cdot 2 \cdot 0 = 0$$

$$\frac{\partial f}{\partial y} = \cos(x^2+y) - \sin(y) \qquad = \underbrace{\cos\left(\frac{\pi}{4}\right)}_{1/\sqrt{2}} - \underbrace{\sin\left(\frac{\pi}{4}\right)}_{1/\sqrt{2}} = 0$$

$$\frac{\partial^2 f}{\partial x^2} = -\sin(x^2+y)\cdot(2x)^2 + \cos(x^2+y)\cdot 2 \qquad = -\sin\left(\frac{\pi}{4}\right)\cdot 0 + \cos\left(\frac{\pi}{4}\right)\cdot 2 = 2/\sqrt{2} = \sqrt{2}$$

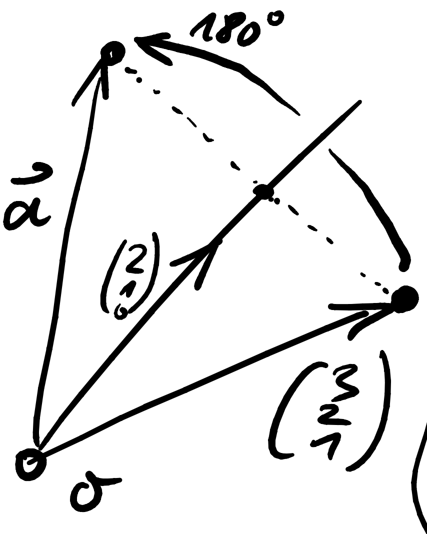
$$\frac{\partial^2 f}{\partial y^2} = -\sin(x^2+y) - \cos(y) \qquad = -\sin\left(\frac{\pi}{4}\right) - \cos\left(\frac{\pi}{4}\right) = -1/\sqrt{2} - 1/\sqrt{2} = -\sqrt{2}$$

$$\frac{\partial^2 f}{\partial x \partial y} = -\sin(x^2+y)\cdot 2x \qquad = 0$$

Hesse-Matrix: $\begin{pmatrix} \sqrt{2} & 0 \\ 0 & -\sqrt{2} \end{pmatrix}$

det ./. $< 0 \Rightarrow$ Sattelpunkt, **kein** lok. Min/Max.

7)



Zerlege: $\begin{pmatrix}3\\2\\1\end{pmatrix} = \vec{b} + \vec{c}$

$\lambda\begin{pmatrix}2\\1\\0\end{pmatrix}$ $\perp \begin{pmatrix}2\\1\\0\end{pmatrix}$

Dann ist $\vec{a} = \vec{b} - \vec{c}$.

Bestimmung von $\vec{b}$ und $\vec{c}$:

$$\underbrace{\begin{pmatrix}2\\1\\0\end{pmatrix}\cdot\begin{pmatrix}3\\2\\1\end{pmatrix}}_{8} = \underbrace{\begin{pmatrix}2\\1\\0\end{pmatrix}\cdot\lambda\begin{pmatrix}2\\1\\0\end{pmatrix}}_{5\lambda} + \underbrace{\begin{pmatrix}2\\1\\0\end{pmatrix}\cdot\vec{c}}_{0}$$

$$\Rightarrow \lambda = 8/5$$

$$\Rightarrow \vec{b} = \frac{8}{5}\begin{pmatrix}2\\1\\0\end{pmatrix}, \quad \vec{c} = \begin{pmatrix}3\\2\\1\end{pmatrix} - \frac{8}{5}\begin{pmatrix}2\\1\\0\end{pmatrix}$$

$$\Rightarrow \vec{a} = \frac{8}{5}\begin{pmatrix}2\\1\\0\end{pmatrix} - \begin{pmatrix}3\\2\\1\end{pmatrix} + \frac{8}{5}\begin{pmatrix}2\\1\\0\end{pmatrix}$$

$$= \frac{16}{5}\begin{pmatrix}2\\1\\0\end{pmatrix} - \begin{pmatrix}3\\2\\1\end{pmatrix}$$

$$\left(= \begin{pmatrix}17/5\\6/5\\-1\end{pmatrix} \right)$$

8) z.B.

$+\begin{pmatrix}3\\3\\3\end{pmatrix}$ $+\begin{pmatrix}3\\3\\3\end{pmatrix}$

$$\begin{pmatrix} 1 & 4 & 7 & 2 \\ 2 & 5 & 8 & 4 \\ 3 & 6 & 9 & 6 \end{pmatrix}$$

$\cdot 2$

Zwei linear unabhängige Vektoren

9) Ansatz: $x(t) = e^{\lambda t}$

$$\Longrightarrow \lambda^2 + a\lambda + 10 = 0$$

$$\Longrightarrow \lambda = -\frac{a}{2} \pm \sqrt{\left(\frac{a}{2}\right)^2 - 10}$$

Wollen: $\operatorname{Re} \lambda < 0$.

Fall 1: $\left(\frac{a}{2}\right)^2 - 10 \geq 0$

Dann $\operatorname{Re}\lambda < 0 \Longleftrightarrow -\frac{a}{2} + \underbrace{\sqrt{\left(\frac{a}{2}\right)^2 - 10}}_{\leq \frac{|a|}{2}} < 0$

$$\Longleftrightarrow a > 0$$

Fall 2: $\left(\frac{a}{2}\right)^2 - 10 < 0$

Dann $\operatorname{Re}\lambda < 0 \Longleftrightarrow -\frac{a}{2} < 0 \Longleftrightarrow a > 0$

Also: $\boxed{a > 0}$.

10) $$c_0 = \frac{1}{4}\int_0^4 (1-t)\,dt = \frac{1}{4}\left[t - \frac{t^2}{2}\right]_0^4$$

$$= \frac{1}{4}\left(4 - \frac{\cancel{16}^{\,8}}{\cancel{2}} - 0\right) = -1$$

$$c_3 = \frac{1}{4}\int_0^4 e^{-2\pi i 3\frac{t}{4}}(1-t)\,dt$$

$$= \frac{1}{4}\underbrace{\int_0^4 e^{-2\pi i 3\frac{t}{4}}\,1\,dt}_{0} - \frac{1}{4}\int_0^4 e^{-2\pi i 3\frac{t}{4}}\,t\,dt$$

$$\frac{e^{-2\pi i 3\frac{t}{4}}}{-2\pi i\frac{3}{4}} \qquad 1$$

$$= -\frac{1}{4}\left[\frac{e^{-2\pi i 3\frac{t}{4}}}{-2\pi i\frac{3}{4}}\,t\right]_0^4 + \frac{1}{4}\int_0^4 \frac{e^{-2\pi i 3\frac{t}{4}}}{-2\pi i\frac{3}{4}}\,dt$$

$$= -\frac{1}{4}\left(\frac{\overbrace{e^{-2\pi i \cdot 3\frac{4}{4}}}^{1}}{-2\pi i\frac{3}{4}}\cdot 4 - 0\right) + \frac{1}{4}\left[\frac{e^{-2\pi i\cdot 3\frac{t}{4}}}{\left(-2\pi i\cdot\frac{3}{4}\right)^2}\right]_0^4$$

$$= \frac{1}{\pi i\frac{3}{2}} + \frac{1}{4}\,\underbrace{\frac{\overbrace{e^{-2\pi i\, 3\frac{4}{4}}}^{1} - e^0}{4\pi^2\cdot(-1)\cdot\frac{9}{16}}}_{0}$$

$$= \frac{-2i}{3\pi}$$

11) $\frac{7}{s^3+s^2} = \frac{7}{s^2(s+1)} = \frac{A}{s^2} + \frac{B}{s} + \frac{C}{s+1}$

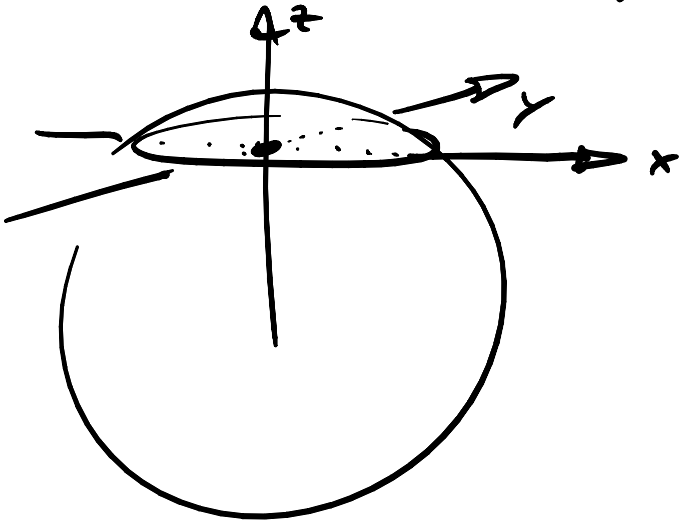
$A = 7$, $C = 7$

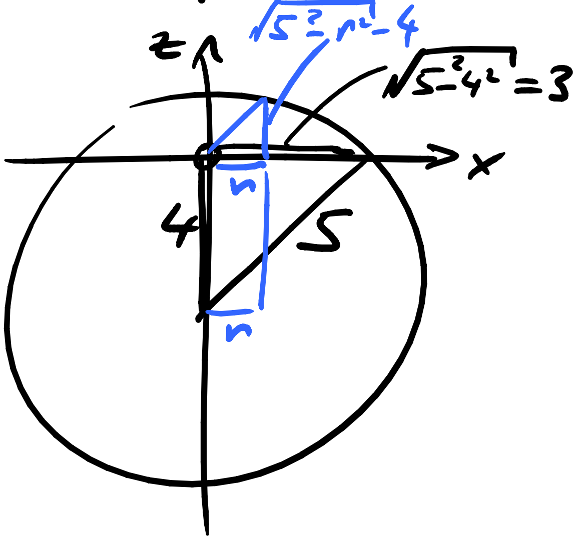
z.B. $\frac{7}{1^3+1^2} \overset{!}{=} \frac{7}{1^2} + \frac{B}{1} + \frac{7}{1+1}$

$\Rightarrow B = \frac{7}{2} - 7 - \frac{7}{2} = -7$

Also Funktion $= 7t - 7 + 7e^{-t}$

12)





$$V = \int_0^3 \left(\int_0^{2\pi} \left(\sqrt{25-r^2}-4\right) d\varphi\right) r\,dr$$

$$= 2\pi\int_0^3 \left(\sqrt{25-\underbrace{r^2}_{u}}-4\right) r\,dr = \cancel{2}\pi\int_0^9 \left(\sqrt{25-u}-4\right)\frac{du}{\cancel{2}}$$

$$\frac{du}{dr} = 2r \Rightarrow r\,dr = \frac{1}{2}du$$

$$= \pi \left[\frac{2}{3}(25-u)^{3/2}\cdot(-1) - 4u\right]_0^9$$

$$= \pi\left(-\frac{2}{3}16^{3/2} - 4\cdot 9 + \frac{2}{3}25^{3/2} - 0\right)$$

$$= \pi\left(-\frac{2}{3}\cdot\underbrace{4^3}_{64} - 36 + \frac{2}{3}\cdot 125\right)$$

$$= \pi\left(\frac{2\cdot 61}{3} - 36\right) = \frac{14}{3}\pi$$