1) Ebenengleichung:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix}$$

xy-Ebene ist $z = 0$.

Also $0 = 1 + 2\mu \Rightarrow \mu = -1/2$

λ beliebig

$\Rightarrow$ Schnittmenge ist

$$\left\{ \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} - \frac{1}{2} \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix}, \lambda \in \mathbb{R} \right\}$$

$$= \left\{ \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \lambda \in \mathbb{R} \right\}$$

2) $\begin{vmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 0 \\ 0 & 3 & 4 & 0 \\ 0 & 5 & 0 & 0 \end{vmatrix}$ $\cdot(-1)$, $\cdot(-3/5)$ $= \begin{vmatrix} 0 & 0 & 0 & 4 \\ 1 & 2 & 3 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 5 & 0 & 0 \end{vmatrix}$ ...

$$= \begin{vmatrix} 0 & 0 & 0 & 4 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 5 & 0 & 0 \end{vmatrix} = +1 \cdot 4 \cdot 4 \cdot 5 = 80$$

vertauschen

3) λ Eigenwert $\Leftrightarrow 0 = \begin{vmatrix} \lambda - 1 & 1 \\ -2 & \lambda - 1 \end{vmatrix}$

$$= \lambda^2 - 2\lambda + 1 + 2$$

$$\lambda = 1 \pm \sqrt{1-3} = 1 \pm \sqrt{2}\, i$$

4) Ansatz: $y = A e^{-x}$

$$\Rightarrow A e^{-x} - 4 A e^{-x} = e^{-x}$$

$$\Rightarrow -3A = 1 \Rightarrow A = -\frac{1}{3}$$

((Probe: $y = -\frac{1}{3} e^{-x}$ einsetzen ✓))

5) f ist ungerade $\Rightarrow$ Alle a sind 0.

$$b_3 = \frac{2}{4} \int_0^4 \underbrace{(t-2)}_{\downarrow\ 1} \underbrace{\sin\left(\cancel{2}\pi \cdot 3 \cdot t/\cancel{4}^{\,2}\right)}_{\uparrow\ -\frac{\cos(\cdot/\cdot)}{3\pi/2}} dt$$

partielle Integration

$$= \frac{1}{2}\left(\underbrace{\left[(t-2) \frac{-\cos(\cdot/\cdot)}{3\pi/2} \right]_0^4}_{-\cancel{2}\,4\frac{\cos(6\pi)}{3\pi/\cancel{2}} - \cancel{2}\,4\frac{\cos(0)}{3\pi/\cancel{2}}} - \underbrace{\int_0^4 \frac{-\cos(\cdot/\cdot)}{3\pi/2}\, dt}_{\frac{1}{3\pi/2}\left[\frac{\sin(\cdot/\cdot)}{3\pi/2}\right]_0^4} \right)$$

$$= \frac{1}{2}\left(-\frac{\cancel{1}\,4}{3\pi} - \frac{\cancel{1}\,4}{3\pi} + \ldots (0-0) \right) = -\frac{4\,\cancel{2}\,\cancel{1}}{3\pi}$$

6) $$\frac{\partial f}{\partial x} = \frac{1}{2\sqrt{x}} \sin(y), \quad \frac{\partial f}{\partial y} = \sqrt{x} \cos(y)$$

$$\frac{\partial f}{\partial x}(2,\pi) = \frac{1}{2\sqrt{2}} \sin(\pi) = 0$$

$$\frac{\partial f}{\partial y}(2,\pi) = \sqrt{2} \cos(\pi) = -\sqrt{2}$$

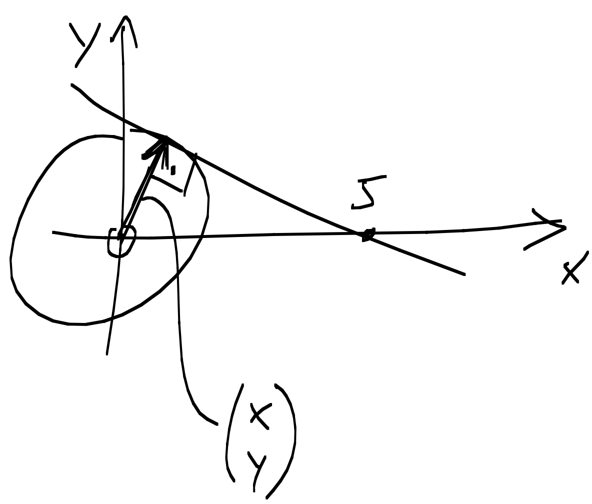
Also Tangentialebene

$$z = \sqrt{2} \underbrace{\sin(\pi)}_{0} - \sqrt{2}(y-\pi)$$

Also Näherung

$$\sqrt{2{,}01} \sin(\pi + 0{,}02) \approx -\sqrt{2} \cdot 0{,}02$$

7)



- $x^2 + y^2 = 2^2 = 4$
- $\underbrace{\binom{x}{y} \cdot \left(\binom{5}{0} - \binom{x}{y} \right)}_{5x \underbrace{-x^2 - y^2}_{-4}} = 0$

$$\Rightarrow x = 4/5 \Rightarrow y = \pm \sqrt{\underbrace{4 - (4/5)^2}_{\frac{84}{25}}} = \pm \frac{\sqrt{84}}{5}$$

$\Rightarrow$ Gleichung für obere Tangente:

$$\binom{x}{y} = \binom{5}{0} + \lambda \begin{pmatrix} \frac{4}{5} - 5 \\ \sqrt{84}/5 \end{pmatrix} = \binom{5}{0} + \frac{\lambda}{5} \begin{pmatrix} -21 \\ \sqrt{84} \end{pmatrix}$$

8) Die drei Vektoren liegen in einer Ebene, weil sie linear abhängig voneinander sind.

Aus der Ebene heraus zeigt z.B.

$$\begin{pmatrix}0\\1\\2\end{pmatrix} \times \begin{pmatrix}1\\0\\-1\end{pmatrix} = \begin{pmatrix}1\cdot(-1) - 2\cdot 0\\ 2\cdot 1 - 0\cdot(-1)\\ 0\cdot 0 - 1\cdot 1\end{pmatrix} = \begin{pmatrix}-1\\2\\-1\end{pmatrix}.$$

9) Ansatz: $y = e^{\lambda x} \Longrightarrow \lambda^2 + \lambda + 1 = 0$

$$\Longrightarrow \lambda_{1,2} = -\frac{1}{2} \pm \sqrt{\frac{1}{4} - 1}$$

$$= -\frac{1}{2} \pm \frac{\sqrt{3}}{2} i$$

Allgemeine Lösung: $y(x) = Ae^{\lambda_1 x} + Be^{\lambda_2 x}$

Das fällt immer exponentiell ab, weil $\mathrm{Re}(\lambda_1) < 0$ und $\mathrm{Re}(\lambda_2) < 0$.

10) $(y')^3 = y \Longleftrightarrow y' = \sqrt[3]{y}$

$$\Longleftrightarrow \frac{y'}{\sqrt[3]{y}} = 1$$

$$\Longrightarrow \underbrace{\int_5^{y_1} \frac{dy}{\sqrt[3]{y}}}_{} = \int_3^{x_1} 1\cdot dx = x_1 - 3$$

$$\left[\frac{3}{2} y^{2/3}\right]_5^{y_1} = \frac{3}{2}\left(y_1^{2/3} - 5^{2/3}\right)$$

$\Downarrow$

$$\Rightarrow \quad y_1 = \left(5^{2/3} + \frac{2}{3}(x_1 - 3)\right)^{3/2}$$

11) $$V = \int_0^{\sqrt{\pi/2}} r\,dr \int_0^{2\pi} d\varphi \, \cos(\underbrace{x^2+y^2}_{r^2})$$

$$= 2\pi \int_0^{\sqrt{\pi/2}} \cos(r^2)\, r\, dr$$

$$r^2 = u, \quad du = 2r\,dr$$

$$\downarrow$$

$$= \pi \int_0^{\pi/2} \cos(u)\, du = \pi \left[\sin(u)\right]_0^{\pi/2}$$

$$= \pi.$$

12) $\boxed{RE}$ $$\frac{s+1}{s^3+s} = \frac{s+1}{s\cdot(s^2+1)}$$

$$\overset{!}{=} \frac{A}{s} + \frac{Bs+C}{s^2+1} = \frac{A(s^2+1)+Bs^2+Cs}{s\cdot(s^2+1)}$$

$$\Rightarrow \quad As^2 + A + Bs^2 + Cs = s+1 \quad \forall s$$

$$\Rightarrow \left\{\begin{matrix} A+B=0 \\ C=1 \\ A=1 \end{matrix}\right\} \Rightarrow \left\{\begin{matrix} A=1 \\ B=-1 \\ C=1 \end{matrix}\right.$$

$$\Rightarrow \quad \frac{s+1}{s^3+s} = \frac{1}{s} + \frac{-s+1}{s^2+1} = \frac{1}{s} - \frac{s}{1+s^2} + \frac{1}{1+s^2}$$

$\Rightarrow$ Originalfunktion $= 1 - \cos(t) + \sin(t)$

$\boxed{\text{ET}}$ $\quad e^{z} = \underbrace{1+j}_{\substack{\text{Länge } \sqrt{2} \\ \text{Winkel } 45^\circ}} = \sqrt{2}\, e^{j\pi/2}$

$a + bj$

$e^{a} \cdot (\cos b + j \sin b)$

Länge $\sqrt{2}$
Winkel 45°

Also $e^{a} = \sqrt{2} \Rightarrow a = \ln(\sqrt{2}) = \frac{1}{2}\ln(2)$

und $b = \pi/2$ oder $\frac{\pi}{2} + 42\pi$ oder ...