Probeklausur 2

1. Asymptote für $x \to \pm\infty$

von $\quad y = \dfrac{2x^3+8}{x^2+x-12}$:

$$(2x^3+8) : (x^2+x-12) = \boxed{2x-2}$$

$$\begin{array}{l} -(2x^3+2x^2-24x) \\ \hline \quad -2x^2+24x+8 \end{array}$$

Asymptoten-
polynom

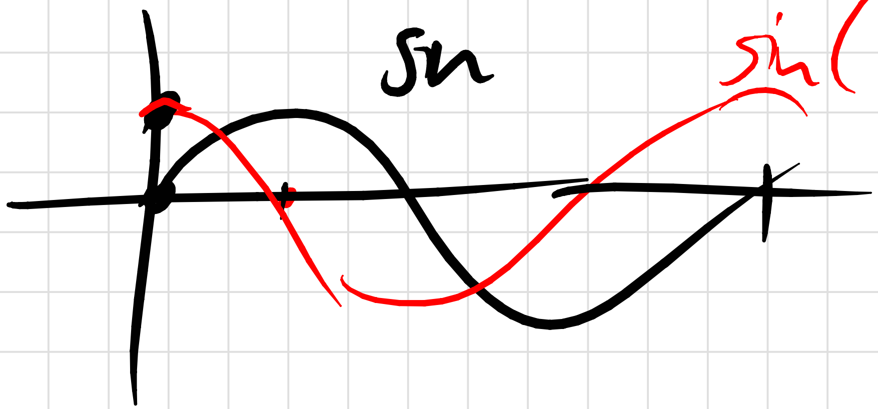

y
x

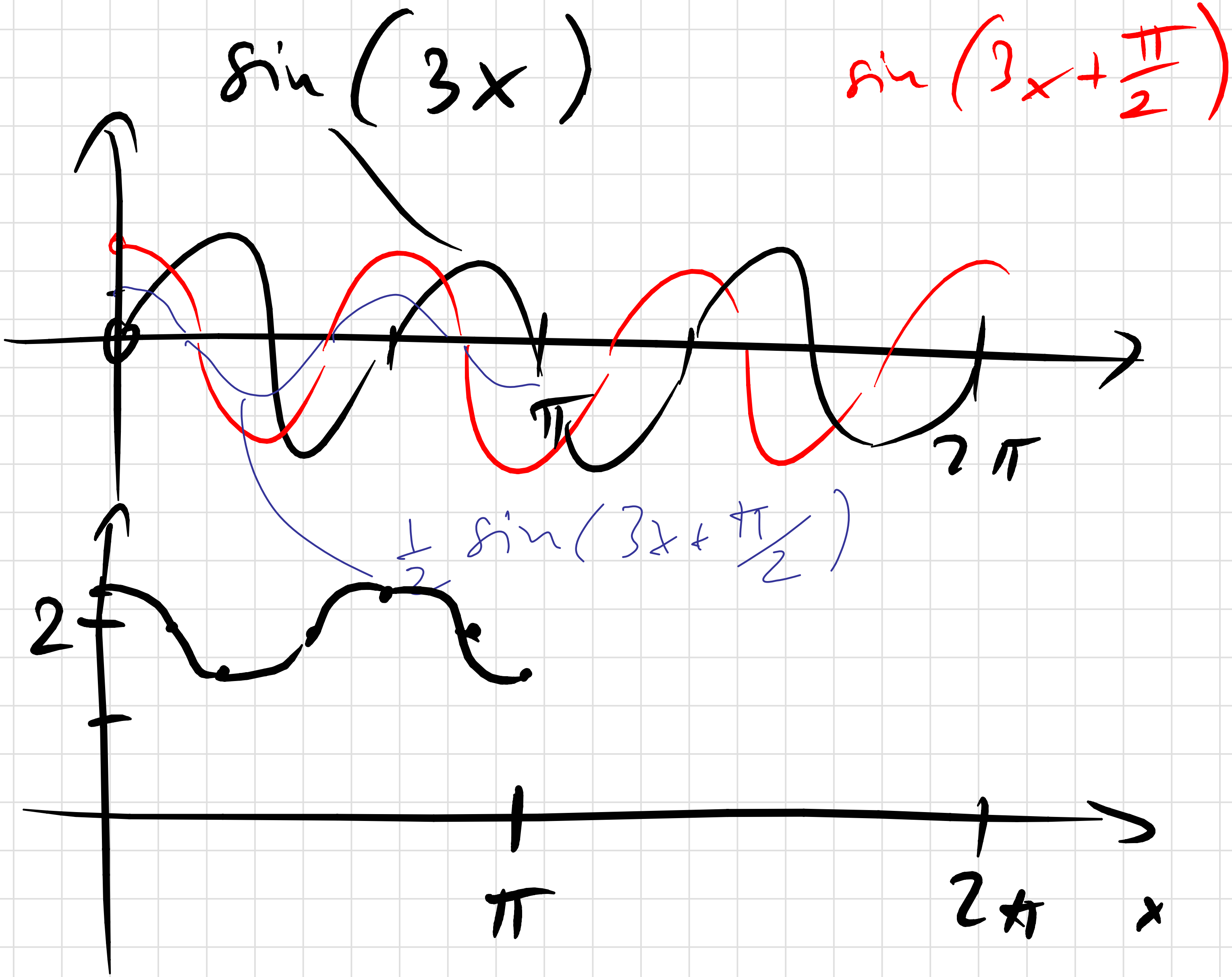
2. $x \mapsto \frac{1}{2}\left(4 + \sin\left(3x + \frac{\pi}{2}\right)\right)$

$= 2 + \frac{1}{2}\sin\left(3x + \frac{\pi}{2}\right)$

$3\,\frac{\pi}{6} + \frac{\pi}{2}$ π

$\sin\left(x + \frac{\pi}{2}\right) =$





sin(3x)
sin(3x + π/2)
1/2 sin(3x + π/2)
π
2π
2
π
2π
x

3. $\dfrac{4+i}{3+2i} = \dfrac{4+i}{3+2i} \cdot \dfrac{3-2i}{3-2i}$

$$= \frac{12-8i+3i+2}{3^2+2^2} = \frac{14}{13} - \frac{5}{13}i$$

4. Wendepunkte von $x \mapsto x^3 - 6x^2 + 6x - 6$:

$$\frac{d\%}{dx} = 3x^2 - 12x + 6$$

$$\frac{d^2\%}{dx^2} = 6x - 12$$

Nulldurchgang bei $x = 2$

$\Rightarrow$ Wendepunkt $\left(2 \;\middle|\; \underbrace{8 \underbrace{\underbrace{-24+12}_{-12}-6}_{-18}}_{-10}\right).$

5. $$x \longmapsto \frac{\ln(x)}{x^2+1}$$

$$\frac{d\%}{dx} = \frac{\frac{1}{x}(x^2+1) - \ln(x)\cdot 2x}{(x^2+1)^2}$$

$$= \frac{x + \frac{1}{x} - 2x\ln(x)}{(x^2+1)^2}$$

6. $$\int_1^3 x\, e^{-x^2}\, dx = \frac{1}{2}\int_1^3 \frac{du}{dx}\, e^{-u}\, dx$$

$$x^2 = u$$

$$\frac{du}{dx} = 2x$$

$$= \frac{1}{2}\int_{1^2}^{3^2} e^{-u}\, du = \frac{1}{2}\left[-e^{-u}\right]_{u=1}^{u=9}$$

$$= \frac{1}{2}\left(-e^{-9} - (-e^{-1})\right)$$

$$= \frac{1}{2}\left(e^{-1} - e^{-9}\right)$$

$$\int_1^3 x\, e^{-x^2}\, dx = \int_{u(1)=1^2}^{u(3)=3^2} \cancel{x}\, e^{-u}\, \frac{1}{2\cancel{x}}\, du$$

$$u = x^2$$

$$= \frac{1}{2} \int_1^9 e^{-u}\, du$$

$$= \ldots$$

$$\frac{du}{dx} = \frac{dx^2}{dx} = 2x$$

$\Rightarrow$ „ $du = 2x\, dx$ "

$\Rightarrow$ „ $dx = \frac{1}{2x}\, du$ "

$$\int_1^3 x\, e^{-x^2}\, dx = \int_{v(1)=-1^2}^{v(3)=-3^2} \cancel{x}\, e^{v} \left(-\frac{1}{2\cancel{x}}\right) dv$$

$$v(x) = -x^2$$

$$\Rightarrow \frac{dv}{dx} = -2x$$

$$\Rightarrow\ dv = -2x\, dx$$

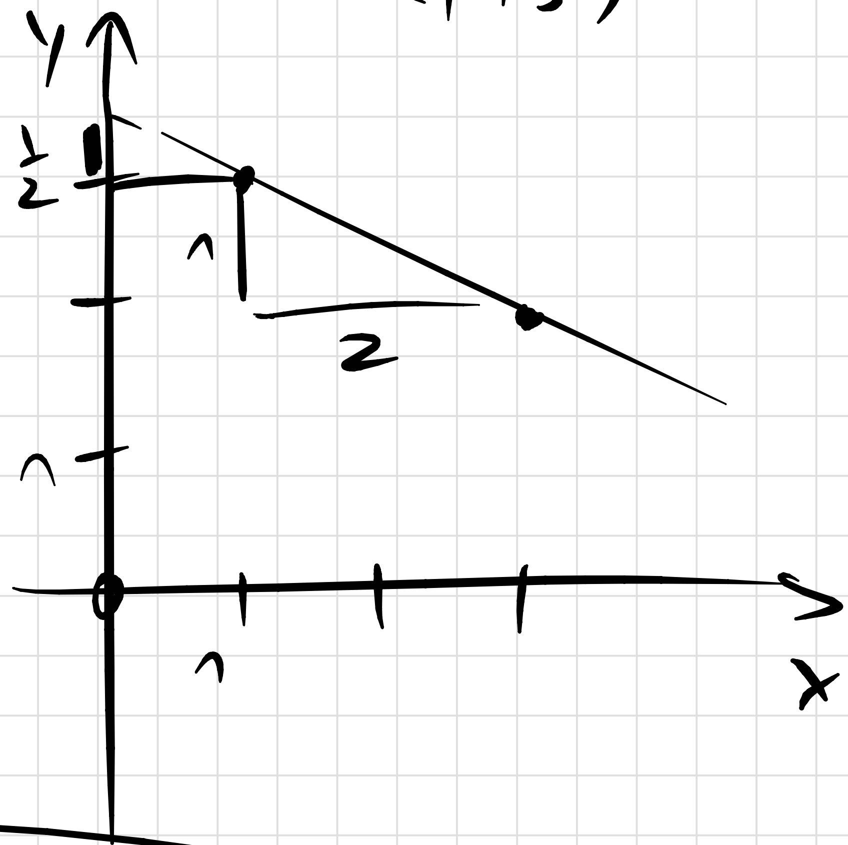
$$\Rightarrow\ dx = -\frac{1}{2x}\, dv$$

$$= -\frac{1}{2} \int_{-1}^{-9} e^{v}\, dv$$

$$= \frac{1}{2} \int_{-9}^{-1} e^{v}\, dv$$

$$= \ldots$$

7. Alle Polynome, die durch (1|3) und (3|2) laufen



$$y = -\frac{1}{2}x + 3\frac{1}{2}$$

+ Polynom mit Nullstellen mindestens bei $x = 1$ und $x = 3$

$$\to (x-1)(x-3) \cdot \text{Polynom}$$

$$= -\frac{1}{2}x + 3\frac{1}{2} + (x-1)(x-3) \cdot \text{Polynom}$$

8. $x \longmapsto x+1 - \dfrac{12}{x+2}$

Nullstellen ?

$$= \frac{(x+1)(x+2) - 12}{x+2}$$

$$= \frac{x^2 + 3x + 2 - 12}{x+2}$$

$$= \frac{x^2 + 3x - 10}{x+2}$$

Nullstellen des Zählers?

$$x^2 + 3x - 10 = 0$$

$$\Rightarrow x = -\frac{3}{2} \pm \underbrace{\sqrt{\underbrace{\frac{9}{4}+10}_{\frac{49}{4}}}}_{\frac{7}{2}}$$

$$\Rightarrow x = 2 \lor x = -5$$

Beides keine Nullstellen des Nenners!

$\Rightarrow$ Beides Nullstellen der gesamten Funktion.

$$\left(\frac{\cancel{(x-2)}(x+5)}{(x-2)^{\cancel{13}\,12}} \quad x=2: \text{ Polstelle} \right)$$

$$\left(\frac{(x-2)^{\cancel{7}}(x+5)}{\cancel{x-2}} \quad x=2: \text{ Nullstelle} \right)$$

$$x+1-\frac{12}{x+2} \overset{!}{=} 0$$

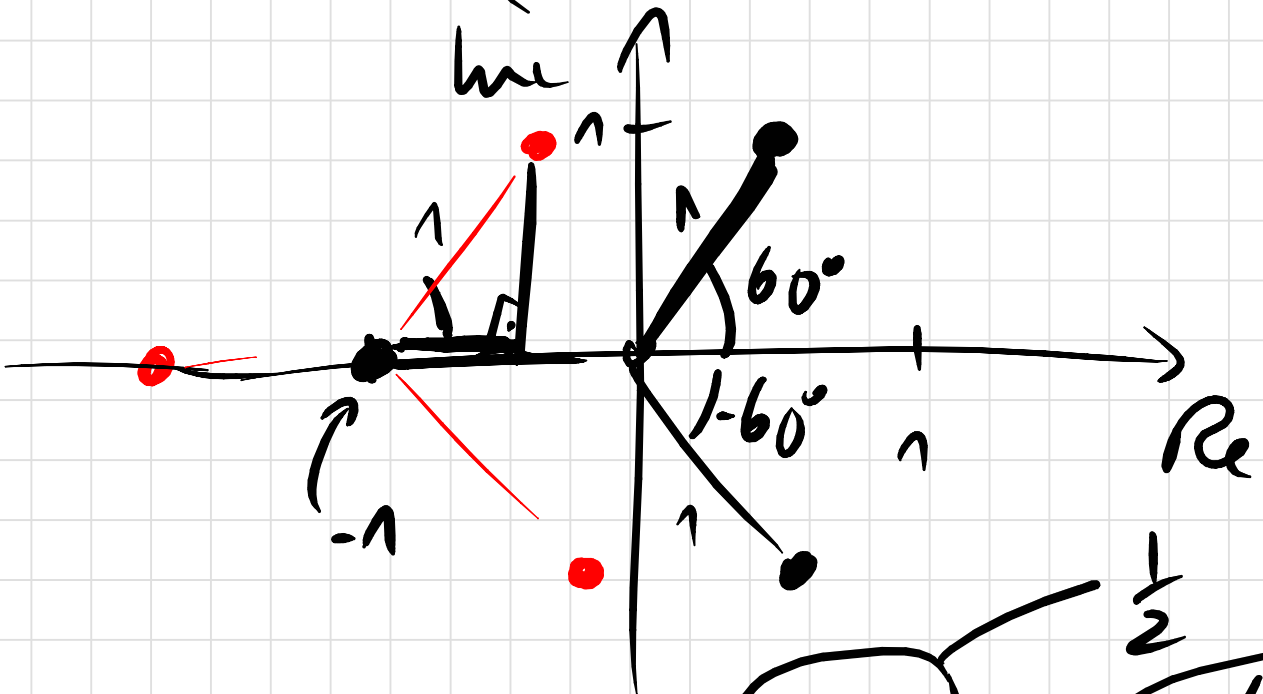
$$\Leftrightarrow \quad x+1 \overset{!}{=} \frac{12}{x+2}$$

Wenn $x \neq -2$:

$$\Rightarrow \quad (x+1)(x+2) = 12 \Rightarrow \ldots$$

9. Finde alle $z \in \mathbb{C}$

mit $(z+1)^3 = -1$!



$$\mathbb{L} = \{-2,\ -1 + \underbrace{\cos(60°)}_{\frac{1}{2}} + i\,\underbrace{\sin(60°)}_{\frac{\sqrt{3}}{2}},$$

$-1 + \cos(60°) - i\sin(60°) \}$

$$z^2 + 3iz + 42 = 0$$

$$\Rightarrow z = -\frac{3i}{2} \pm \sqrt{-\frac{9}{4} - 42}$$

$$= \ldots$$

10. $x \longmapsto x^2(1-x) = x^2 - x^3$

$\max\limits_{x \in [4,5]} (\%) = ?$

$\frac{d\%}{dx} = 2x - 3x^2$

notwendig für lok. Max:

$2x - 3x^2 = 0 \Longleftrightarrow 2x = 3x^2$

$$\iff x = 0 \lor \underbrace{2 = 3x}_{x = 2/3}$$

In $[4;5]$ liegt **kein** lok. Max.

$\Rightarrow$ max Wert am Rand

$$\Rightarrow \max = \max(-16 \cdot 3, -25 \cdot 4)$$

$$= -16 \cdot 3 = -48$$

11. $$\int_0^1 \tan(\varphi)\, d\varphi = \int_0^1 \frac{\sin(\varphi)}{\cos(\varphi)}\, d\varphi$$

$$= -\int_{\cos(0)}^{\cos(1)} \frac{1}{u}\, du$$

$$u = \cos(\varphi)$$

$$\frac{du}{d\varphi} = -\sin(\varphi)$$

$$\Longrightarrow \text{,, } du = -\sin(\varphi)\, d\varphi \text{ ''}$$

$$= -\Big[\ln|u|\Big]_{\cos(0)}^{\cos(1)}$$

$$= -\ln|\cos(1)| - \left(-\ln|\cos(0)|\right)$$

$$= -\ln|\cos(1)| \cancel{+0}.$$

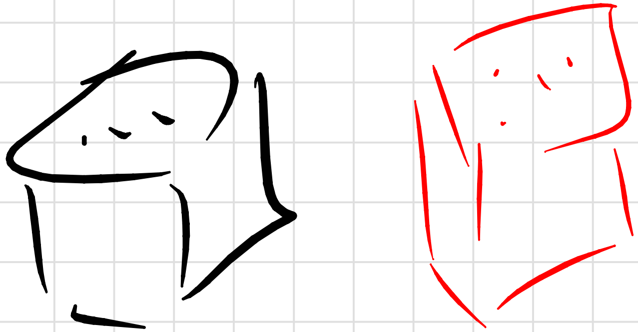
12. Zwei Würfel,

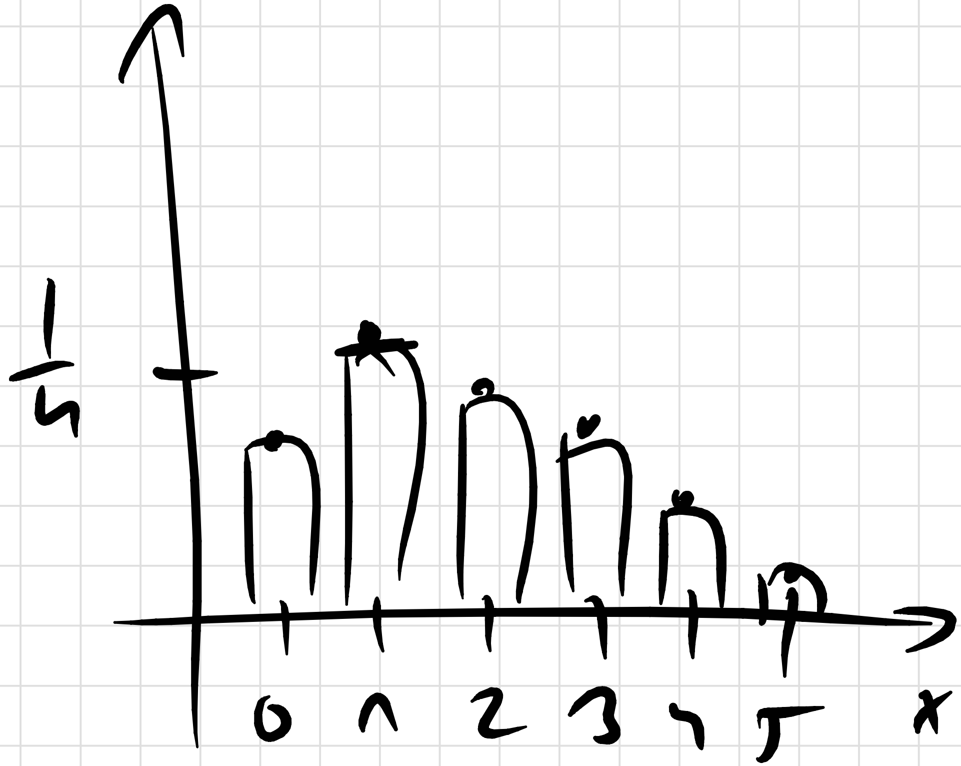
X = Differenz der Augenzahlen

0 oder positiv !

Werte : 0, 1, 2, 3, 4, 5

$P = \frac{6}{36}, \frac{10}{36}, \frac{8}{36}, \frac{6}{36}, \frac{4}{36}, \frac{2}{36}$





1/4
0
1
2
3
4
5
x