Mathematik 1

28. Januar 2022

Musterlösungen

1. $\left(5+\log_{10}(x)\right)^3 = 8$

$$\iff 5+\log_{10}(x) = \sqrt[3]{8} = 2$$

$$\iff \log_{10}(x) = -3$$

$$\iff x = 10^{-3} = 0{,}001.$$

2. $x-1 > |x-3|$

$$\iff x \geq 3 \wedge x-1 > |x-3|$$
$$\vee\ x<3 \wedge x-1 > |x-3|$$

$$\iff x \geq 3 \wedge x-1 > x-3$$
$$\vee\ x<3 \wedge x-1 > 3-x$$

$\cancel{2}x > \cancel{4}^{\,2}$

$-1 > -3$ immer wahr

$$\iff x \geq 3 \ \vee\ x<3 \vee x>2$$

Also $\mathbb{L} = (2;\infty)$.

3. $(z+3i)^2 = -1$

$\iff z + 3i = i \lor z + 3i = -i$

$\iff z = 0 - 2i \lor z = 0 - 4i$

Realteil　　　Imaginärteil

((oder Quadrat auflösen und dann pq-Formel))

4. $3x^2-6x+3 = 3(x^2-2x+1)$
$= 3(x-1)^2$

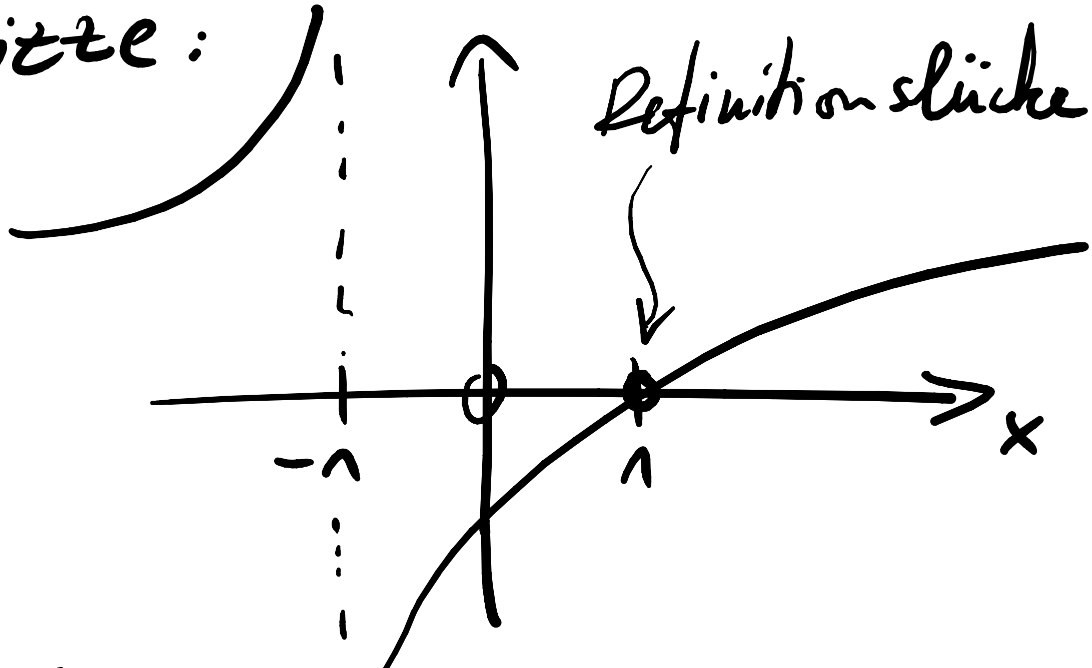
Also:

$$\frac{3x^2-6x+3}{x^2-1} = \frac{3(x-1)^{\cancel{2}}}{(x+1)\cancel{(x-1)}}$$

$$= \frac{3(x-1)}{x+1} = \frac{3x-3}{x+1}$$, aber $D = \mathbb{R}\setminus\{1\}$

Nullstellen: keine ($x=1$ nach stetiger Fortsetzung)
Polstellen: $x=-1$ (1. Ordnung)
Asymptote: $(3x-3):(x+1) = 3$ Rest...

Skizze:




5. $$\frac{d\sin(x^3+5e^x)}{dx}$$

$$= \cos(x^3+5e^x)\cdot(3x^2+5e^x)$$

6. $V = \pi \int_3^5 \sqrt{x}^2 \, dx = \pi \int_3^5 x \, dx$

$= \pi \left[ \frac{x^2}{2} \right]_3^5 = \pi \frac{25-9}{2} = 8\pi$

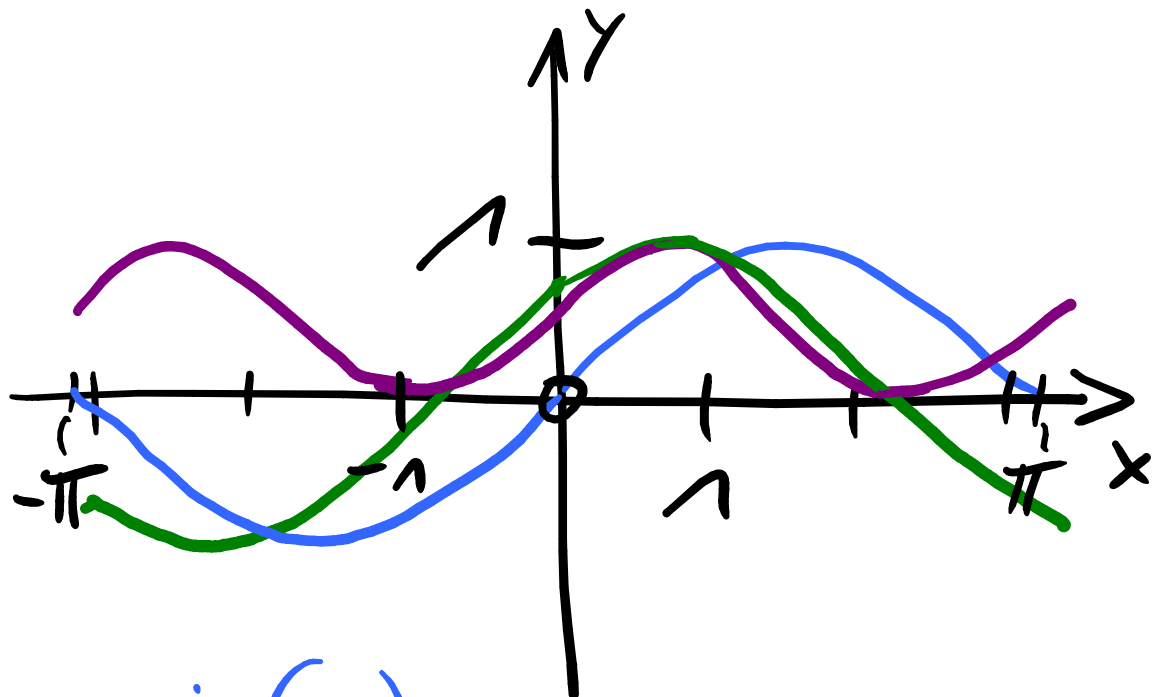
7. $P(\{\text{drei gleicher Farbe}\})$

$= P(\{\text{dreimal rot}\})$

$+ P(\{\text{dreimal grün}\})$

$= \left(\frac{5}{12}\right)^3 + \left(\frac{7}{12}\right)^3$

8.

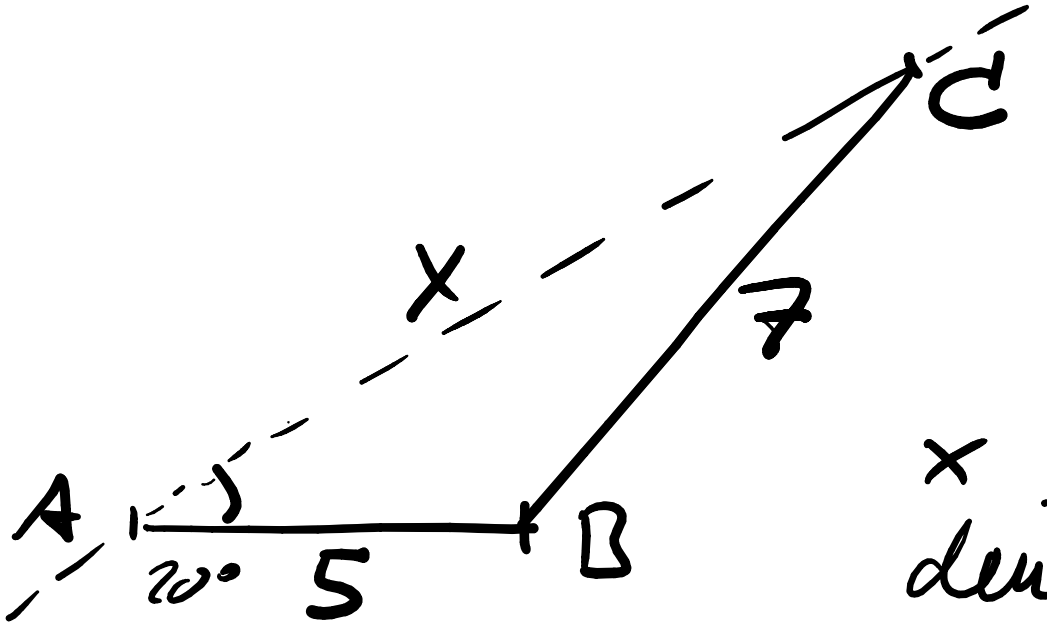



$y = \sin(x)$

$y = \sin\left(x + \frac{\pi}{4}\right)$

$y = \left(\sin\left(x + \frac{\pi}{4}\right)\right)^2$

9.




x ist eindeutig bestimmt!

Cosinussatz:

$$7^2 = x^2 + 5^2 - 2\cdot 5\cdot x\cdot \cos(20°)$$

$$\Rightarrow x^2 - 10\cos(20°)\,x \underbrace{+25-49}_{-24} = 0$$

$$\Rightarrow x = 5\cos(20°) \pm \sqrt{(5\cos(20°))^2 + 24}$$

Minus nicht möglich, weil dann $x<0$.

10. Beobachtung:

- Keine Definitionslücken
- Funktionswert $\to 0$ für $x \to \pm\infty$

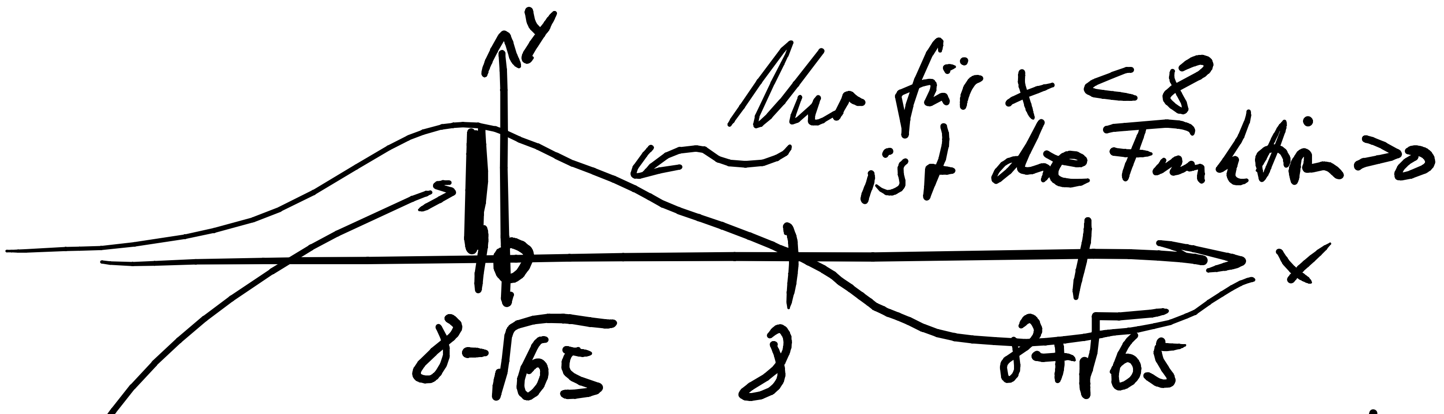
Suche lokale Extrema:

$$0 \overset{!}{=} \frac{d\,\frac{8-x}{x^2+1}}{dx} = \frac{-1\cdot(x^2+1)-(8-x)\cdot 2x}{(x^2+1)^2}$$

$$\iff 0 = \text{Zähler} = -x^2-1-16x+2x^2$$

$$= x^2-16x-1$$

$$\iff x = 8 \pm \sqrt{65}.$$




Dies muss der größte Wert sein!

Der ist $\dfrac{\cancel{8}-\cancel{8}+\sqrt{65}}{(8-\sqrt{65})^2+1} \left(\left(= 4+\dfrac{\sqrt{65}}{2}\right)\right).$

11.
$$\int_4^5 (\ln(x))^2 \underbrace{\frac{1}{x}\,dx}_{du} = \int u^2\,du$$

$u = \ln(5)$
$x = 5$

$x = 4$
$u = \ln(4)$

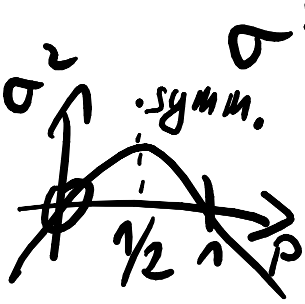
$u = \ln(x)$

$$\frac{du}{dx} = \frac{1}{x}$$

$$= \left[\frac{u^3}{3}\right]_{\ln(4)}^{\ln(5)} = \frac{(\ln(5))^3 - (\ln(4))^3}{3}$$

12. $P[\{X=1\}] =: p$

$$\Rightarrow E[X] = p \cdot 1 + (1-p) \cdot 2 = 2 - p,$$
$$E[X^2] = p \cdot 1^2 + (1-p) \cdot 2^2 = 4 - 3p,$$
$$\sigma^2 = E[X^2] - (E[X])^2 = 4 - 3p - (2-p)^2$$
$$= \cancel{4} - 3p - \cancel{4} + 4p - p^2 = p - p^2$$




ist max. $\frac{1}{2} - \left(\frac{1}{2}\right)^2 = \frac{1}{4}$.