# Mathematik I

Klausur vom 2020-01-31

Musterlösungen

1. $\sqrt{10^{5+x}+9} = 4 \iff 10^{5+x}+9=16$

$\iff 10^{5+x} = 7 \iff 5+x = \log_{10}(7)$

$\iff x = \log_{10}(7) - 5$

2. $(x^3 + 7x^2) : (x^2+1) = \underbrace{x + 7}_{\text{Asymptote}}$ Rest ...

$$\begin{array}{l} -(x^3 + x) \\ \hline 7x^2 - x \\ -(7x^2+7) \\ \hline \dots \end{array}$$

3. $\underbrace{(1-2i)}_{\substack{\text{Länge: } \sqrt{1^2+2^2} = \sqrt{5} \\ \text{Winkel: } -\arctan(2/1)}} z^3 = 1$

Also $z = \frac{1}{\sqrt[6]{5}} e^{i\frac{1}{3}\arctan(2)}$

$= \frac{1}{\sqrt[6]{5}} \cos\left(\frac{\arctan(2)}{3}\right) + \frac{1}{\sqrt[6]{5}} \sin(\text{\%})\, i$

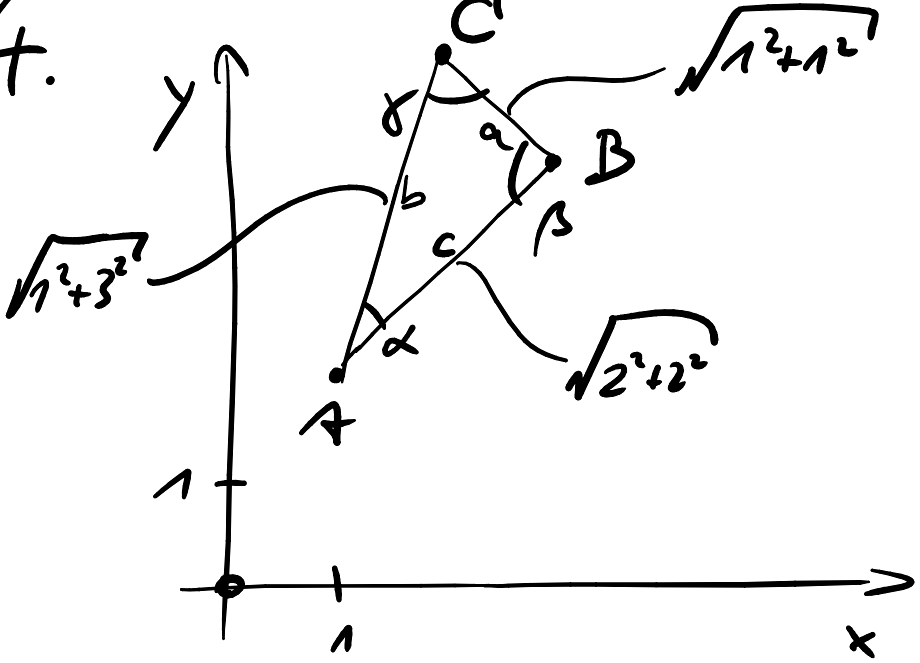
oder $z = \frac{1}{\sqrt[6]{5}} e^{i\frac{1}{3}\arctan(2) + i\frac{2\pi}{3}}$

$= \frac{1}{\sqrt[6]{5}} \cos\left(\frac{\arctan(2)}{3} + \frac{2\pi}{3}\right) + \frac{1}{\sqrt[6]{5}} \sin(\text{\%})\, i$

oder $z = \frac{1}{\sqrt[6]{5}} e^{\cdots\cdots\cdots \frac{4\pi}{3}}$

$= \cdots\cdots \frac{4\pi}{3} \cdots\cdots \frac{4\pi}{3}$

4.



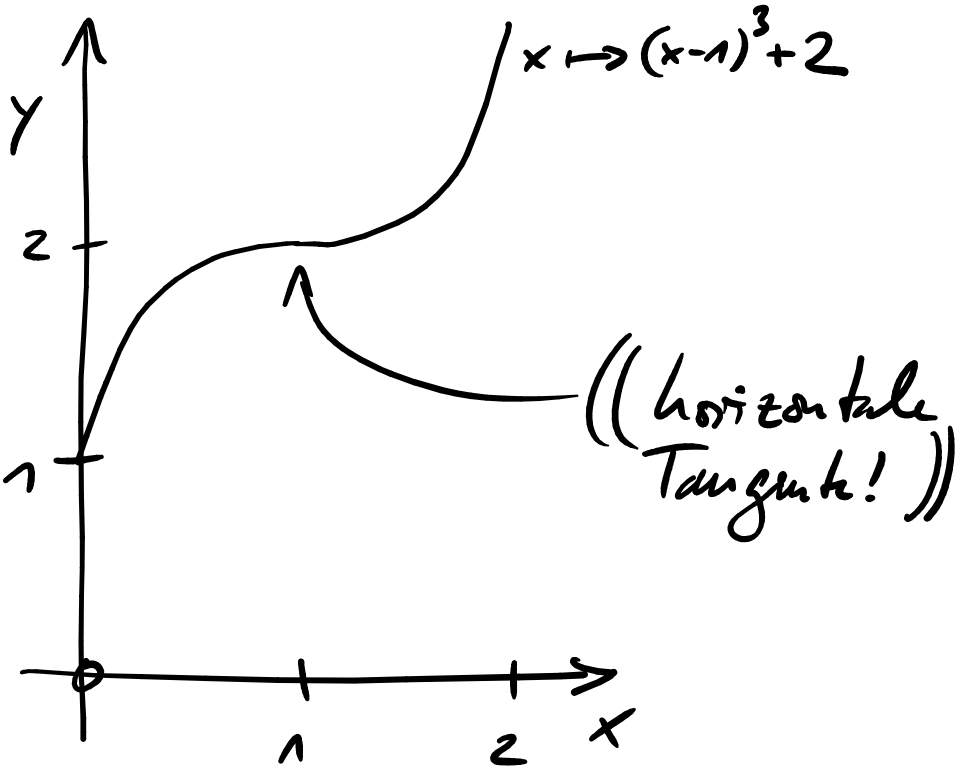

((Oder noch viel einfacher, indem man merkt, dass $\beta = 90°$ ist!))

$$\underbrace{a^2}_{2}+\underbrace{b^2}_{10}-2\underbrace{ab}_{\sqrt{20}}\cos(\gamma)=\underbrace{c^2}_{8}$$

$$\Longrightarrow \gamma = \arccos\left(\frac{\overset{2}{\cancel{2+10-8}}}{\cancel{2}\cdot\sqrt{20}}\right)$$

$$((\approx 63°))$$

5.



6. $$\underbrace{\int_1^4 c e^x \, dx}\overset{!}{=} 1$$

$$c\,[e^x]_1^4 = c\cdot(e^4 - e^1)$$

$$\Rightarrow c = \frac{1}{e^4-e}$$

$$E[X] = \int_1^4 x \cdot \frac{1}{e^4-e} e^x \, dx$$

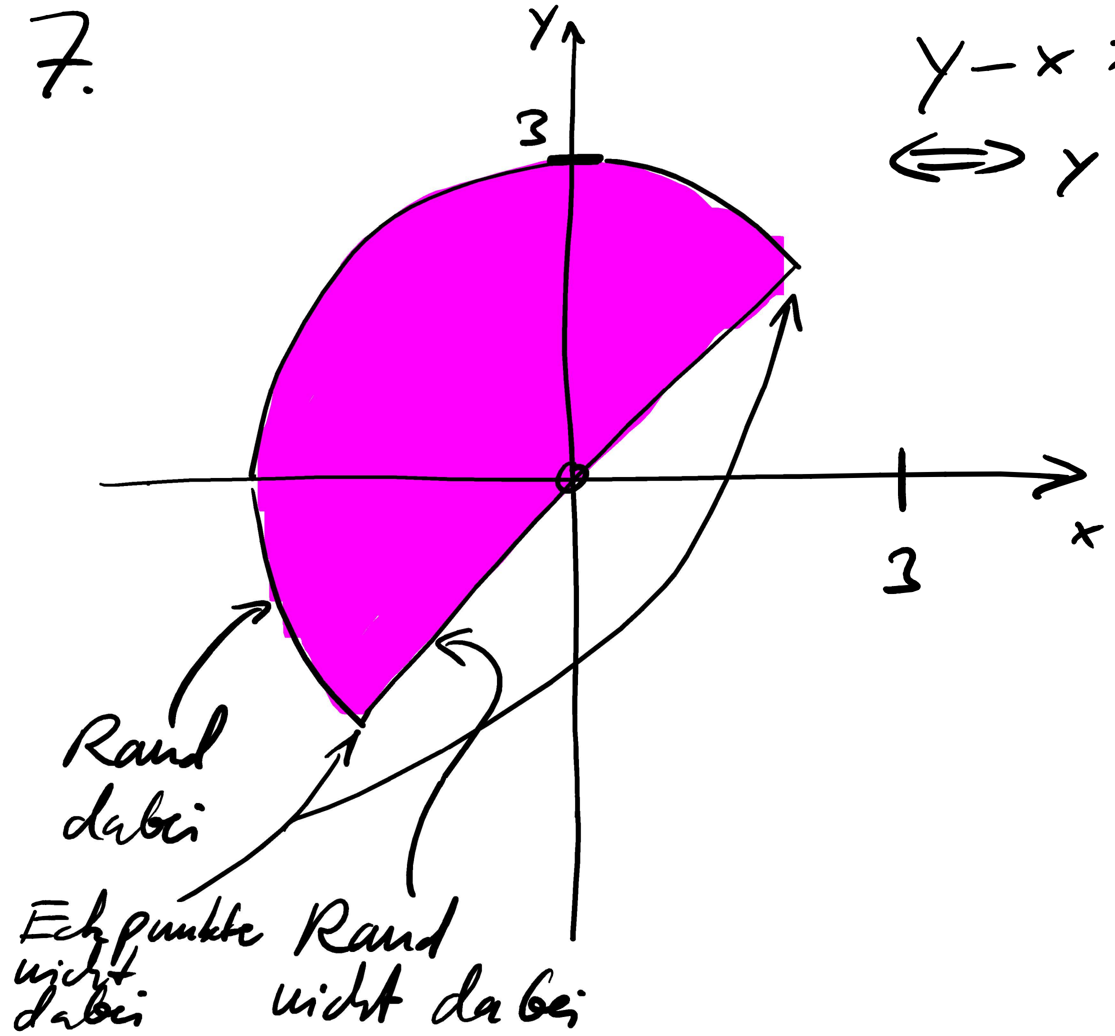
$$= \frac{1}{e^4-e} \int_1^4 \underset{\downarrow 1}{x}\,\underset{\uparrow e^x}{e^x} \, dx$$

$$= \frac{1}{e^4-e}\left(\underbrace{[xe^x]_1^4}_{4e^4-e} - \underbrace{\int_1^4 1\cdot e^x \, dx}_{[e^x]_1^4 = e^4-e}\right)$$

$$= \frac{1}{e^4-e}\left(4e^4 \cancel{-e} - e^4 \cancel{+e}\right) = \frac{3e^4}{e^4-e}$$

$$= \frac{3}{1-e^{-3}} \quad ((\approx 3))$$

7.




$$y - x > 0$$
$$\iff y > x$$

8. Nullstellen? $z^4 + 6z^2 + 5 = 0$

$$\iff z^2 = -3 \pm \sqrt{9-5}$$
$$= -3 \pm 2$$
$$\iff z^2 = -5 \lor z^2 = -1$$

Also $p(z) = (z - \sqrt{5}i)(z + \sqrt{5}i)(z + i)(z - i)$.

9. ((Dies ist die Definition der Ableitung von $x \mapsto x^3$ an der Stelle $x = 2$.
Oder mit L'Hospital.))
Der Grenzwert existiert und ist gleich
$3 \cdot 2^2 = 12$.

10. $f(x) = \dfrac{1}{\frac{1}{3}x^3 - 2x^2 + 3x - 5}$

$$f'(x) = -\frac{1}{(\ldots)^2}\cdot\left(x^2 - 4x + 3\right)$$

$$f''(x) = 2\cdot\frac{1}{(\ldots)^3}\cdot\left(x^2 - 4x + 3\right)^2 - \frac{1}{(\ldots)^2}\cdot(2x - 4)$$

$$f'(x) \overset{!}{=} 0$$

$$\Leftrightarrow x^2 - 4x + 3 = 0 \Leftrightarrow x = 2 \pm \sqrt{4-3}$$

$$= 2 \pm 1$$

$$\Leftrightarrow x = 1 \lor x = 3$$

$$f''(1) = \cancel{2}\cdot 0 - \underbrace{\frac{1}{(\ldots)^2}}_{>0}\cdot\underbrace{(2\cdot 1 - 4)}_{<0}$$

$>0$

$$f''(3) = \cancel{2}\cdot 0 - \underbrace{\frac{1}{(\ldots)^2}}_{>0}\cdot\underbrace{(2\cdot 3 - 4)}_{>0}$$

$<0$

Also genau ein lok. Max.,
nämlich bei $x = 3$.
((Oder mit lok. Min. des Nenners.))

11.

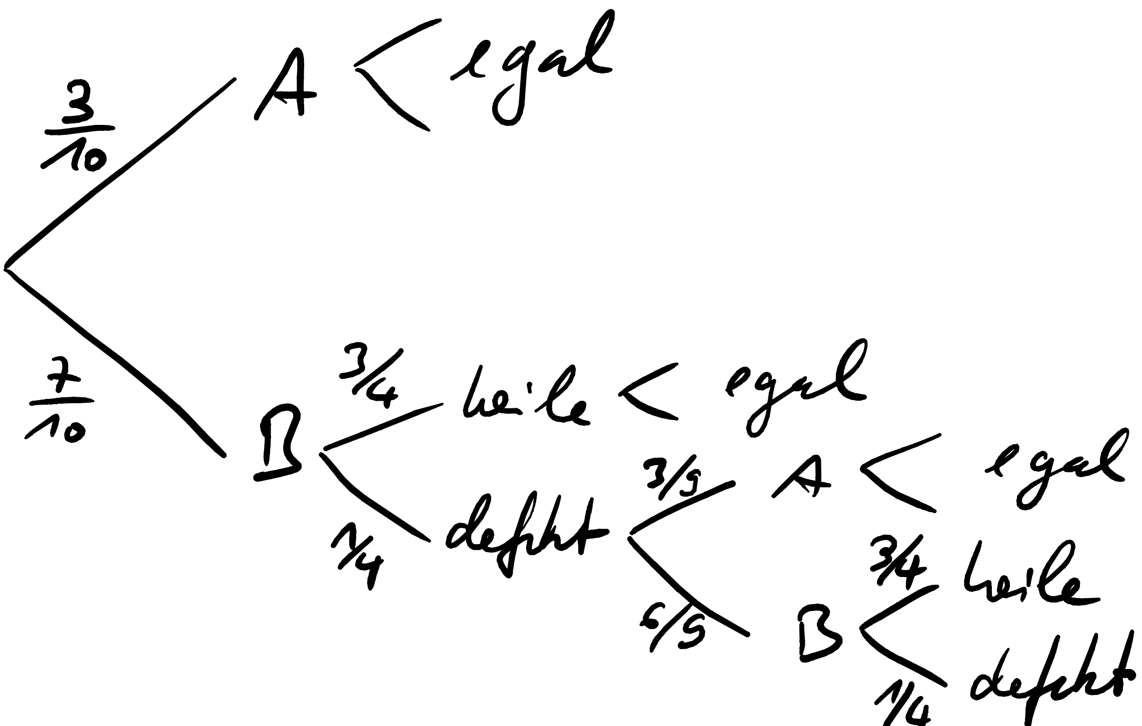
$$\int_0^{\frac{1}{2}} \sin(2x)\cos(2x)\,dx$$

$$\begin{cases} u = \sin(2x) \\ \dfrac{du}{dx} = 2\cdot\cos(2x) \end{cases}$$

$$= \int_{\sin(0)}^{\sin(2\cdot\frac{1}{2})} u \cdot \frac{1}{2}\,\frac{du}{\cancel{dx}}\,\cancel{dx}$$

$$= \frac{1}{2}\left[\frac{u^2}{2}\right]_0^{\sin(1)} = \frac{(\sin(1))^2}{4}$$

12.




Also $P = \frac{7}{10}\cdot\frac{1}{4}\cdot\frac{6}{9}\cdot\frac{1}{4}$

$$\left(\left(= \frac{7\cdot\cancel{6}}{10\cdot 4\cdot \underset{3}{\cancel{9}}\cdot\underset{2}{\cancel{4}}} = \frac{7}{240} \approx 3\,\%\right)\right).$$