Mathematik 1   2018-07-18

Musterlösungen

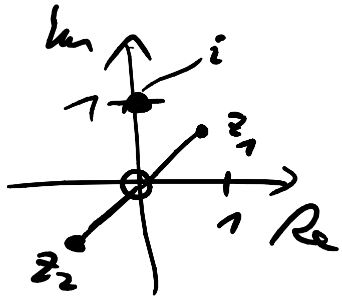
1. $e^{\sqrt{x^2+7}} = 5 \iff \sqrt{x^2+7} = \ln(5)$

$\iff x^2+7 = (\ln(5))^2 \iff x^2 = \underbrace{(\ln(5))^2 - 7}_{<0}$

$\left(\left(\iff x = \pm\sqrt{(\ln(5))^2-7} \notin \mathbb{R}\right)\right)$

$\iff$ Es gibt keine solche reelle Zahl x.

2. $\underbrace{z^3 - iz}_{z(z^2-i)} = 0 \iff z = 0 \lor z^2 = i$



$$\iff \left\{ \begin{array}{l} z = 0 + 0i \\ \lor\ z = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} i \\ \lor\ z = -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} i \end{array} \right.$$

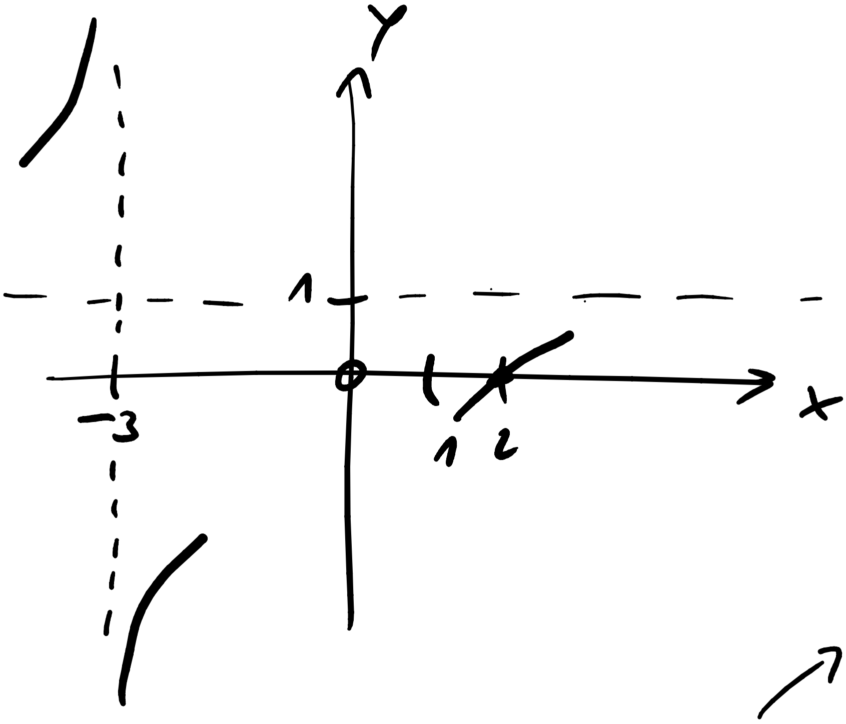
3. Nullstellen des Zählers: $x = 2 \pm \sqrt{4-4} = 2$

" " Nenners: $x = -\frac{1}{2} \pm \sqrt{\underbrace{\frac{1}{4}+6}_{\frac{25}{4}}} = -\frac{1}{2} \pm \frac{5}{2}$

Also $\frac{x^2-4x+4}{x^2+x-6} = \frac{(x-2)^{\cancel{2}}}{\cancel{(x-2)}(x+3)}$.

Also ((einfache)) Nullstelle bei $x = 2$ und Polstelle ((erster Ordnung)) bei $x = -3$.

Asymptote für $x \to \pm\infty$ ist ((offensichtlich)) $y = 1$.



4. $$\frac{5+n^3+n^2\sin(n)}{e^{-n}+7n^3} = \frac{\underbrace{\frac{5}{n^3}}_{\to 0}+1+\underbrace{\frac{\sin(n)}{n}}_{\to 0}}{\underbrace{\frac{e^{-n}}{n^3}}_{\to 0}+7}$$

$$\to \frac{1}{7}$$

5. $$\frac{d\sqrt[3]{\frac{\sin(2x)}{x+1}}}{dx}$$

$$= \frac{1}{3}\left(\frac{\sin(2x)}{x+1}\right)^{-2/3}\cdot\frac{\cos(2x)\cdot 2\cdot(x+1)-1\cdot\sin(2x)}{(x+1)^2}$$

6. $$\int_1^3 x e^{x^2}\,dx = \frac{1}{2}\int_1^9 e^u\,du = \frac{1}{2}\left[e^u\right]_1^9 = \cancel{\frac{9-1}{2}=4}$$

$$= \frac{e^9-e}{2}$$

$u = x^2,$
$du = 2x\,dx$

7. $$P = \frac{\binom{6}{3}}{6^3} = \frac{\frac{\cancel{6}\cdot 5\cdot \cancel{4}}{3\cdot \cancel{2}\cdot 1}}{\cancel{6}\cdot 6\cdot \cancel{6}_{3}} = \frac{5}{54}$$

8. $$|x-5| > x^2$$

$$\Longleftrightarrow \quad x-5 \geq 0 \ \wedge \ x-5 > x^2$$
$$\vee \ x-5 < 0 \ \wedge \ -(x-5) > x^2$$

„=" für $x = \underbrace{\frac{1}{2} \pm \sqrt{\frac{1}{4}-5}}_{\text{nie!}}$

immer falsch, weil immer $x^2 > x-5$

„=" für $x = -\frac{1}{2} \pm \sqrt{\frac{1}{4}+5}$

$$\Longleftrightarrow \quad \cancel{x<5} \ \wedge \ x > -\frac{1}{2} - \sqrt{\frac{1}{4}+5} \ \wedge \ x < -\frac{1}{2} + \sqrt{\ldots}$$

Also $$\mathbb{L} = \left(-\frac{1}{2} - \frac{\sqrt{21}}{2}, -\frac{1}{2} + \frac{\sqrt{21}}{2}\right).$$

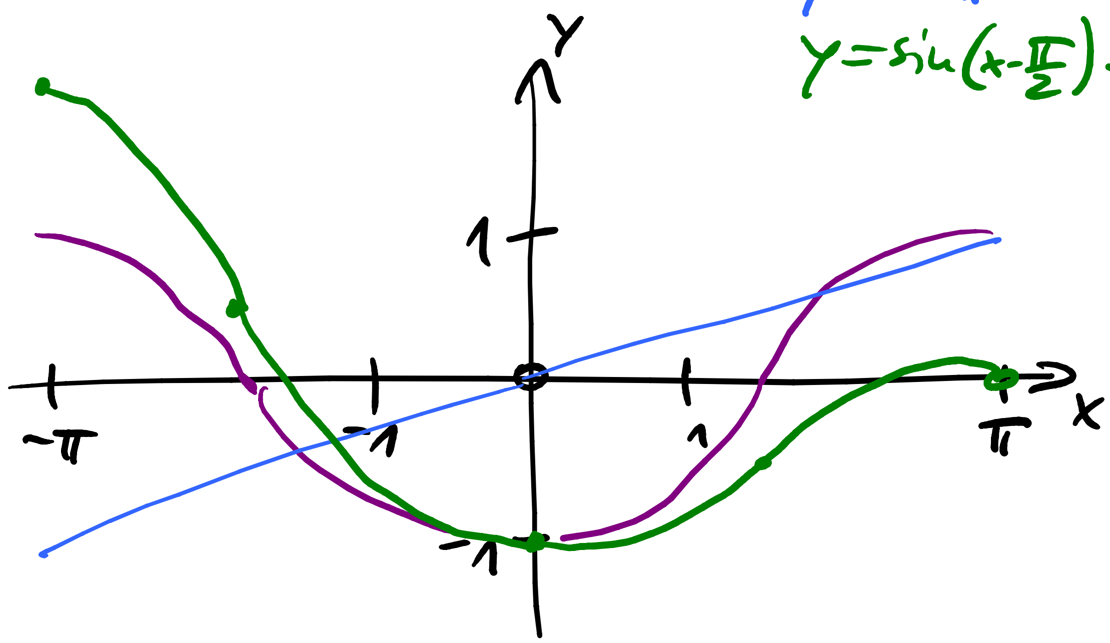
offenes Intervall

9.

$y = \sin\left(x - \frac{\pi}{2}\right)$

$y = x/\pi$

$y = \sin\left(x - \frac{\pi}{2}\right) - \frac{x}{\pi}$



10. [[Beobachtung: $\left|\frac{1+ix}{1-\underset{\text{reell}}{ix}}\right| = \frac{\sqrt{1+x^2}}{\sqrt{1+x^2}} = 1$

Also liegt $f(x)$ immer auf dem Kreis mit Radius 1 um den Ursprung in der Gaußschen Zahlenebene.]]

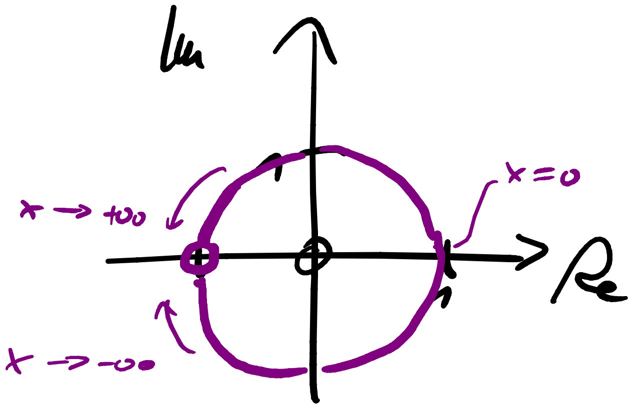


Bild von f ist: Einheitskreis um Ursprung, aber ohne die -1.

11. $$E[X^3] = \underbrace{p}_{P(\{X=1\})}\cdot 1^3 + (1-p)\cdot 2^3$$

$$= p\cdot\underbrace{(1-8)}_{-7} + 8 \overset{!}{=} 5 \Rightarrow p = \frac{3}{7}$$

12. $$V = \pi\int_0^2 (2x-x^2)^2\,dx = \pi\int_0^2 (4x^2-4x^3+x^4)\,dx$$

$$= \pi\left[4\frac{x^3}{3} - \cancel{4}\frac{x^4}{\cancel{4}} + \frac{x^5}{5}\right]_0^2 = \pi\left(\frac{32}{3} - 16 + \frac{32}{5}\right) - 0$$

(($= 32\pi\cdot\left(\frac{1}{3} - \frac{1}{2} + \frac{1}{5}\right) = \cancel{32}^{16}\pi\cdot\frac{10-15+6}{\cancel{30}_{15}} = 16\pi/15$))