Mathematik-2-Klausur vom 2017-07-28
Musterlösungen

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1. λ E.W. $\iff 0 = \begin{vmatrix} 2-\lambda & 1 & 0 \\ 4 & 5-\lambda & 3 \\ 0 & 0 & 1-\lambda \end{vmatrix}$

$$= (2-\lambda)(5-\lambda)(1-\lambda) + 0 + 0$$
$$- 0 - 0 - (1-\lambda)\cdot 4 \cdot 1$$
$$= (1-\lambda)\big((2-\lambda)(5-\lambda) - 4\big)$$

$$\iff \lambda = 1 \lor \underbrace{\underbrace{\underbrace{(2-\lambda)(5-\lambda)}_{\lambda^2 - 7\lambda + 6} - 4 = 0}_{\lambda = \frac{7}{2} \pm \sqrt{\frac{49}{4} - 6}}}_{}$$

$$\underbrace{= \frac{7}{2} \pm \frac{5}{2}}_{\lambda = 6 \ \lor\ \lambda = 1}$$

Also Eigenwerte: 1 und 6.

2. $y' + y\sin(x) = 0 \iff \frac{y'}{y} = -\sin(x)$

$$\iff \frac{dy}{y} = -\sin(x)\,dx \implies \underbrace{\int_3^{y_1} \frac{dy}{y}}_{[\ln|y|]_3^{y_1}} = \underbrace{\int_1^{x_1} (-\sin(x))\,dx}_{[\cos(x)]_1^{x_1}}$$

$$\implies y_1 = 3\,e^{\cos(x_1) - \cos(1)}$$

3. homogene Form: $y' + y = 0$

Ansatz: $y(x) = e^{\lambda x}$

$\Rightarrow \lambda + 1 = 0 \Rightarrow \lambda = -1$

$\Rightarrow$ Allg. Lsg. der homog. Form ist $y_h(x) = A e^{-x}$.

inhomogene Form: $y' + y = -x^2$

Ansatz für spez. Lsg.:

$y(x) = Bx^2 + Cx + D$

$\Rightarrow 2Bx + C + Bx^2 + Cx + D = -x^2$

$\Leftrightarrow B = -1 \wedge C = 2 \wedge D = -2$

$\Rightarrow y_i(x) = -x^2 + 2x - 2$

allgemeine Lösung der DGL:

$y(x) = A e^{-x} - x^2 + 2x - 2$.

4. $f(x) = \cos(\sin(x))$, $f'(x) = -\sin(\sin(x)) \cdot \cos(x)$,

$f''(x) = -\cos(\sin(x)) \cdot \cos(x) \cdot \cos(x) - \sin(\sin(x)) \cdot (-\sin(x))$.

$f(0) = 1$, $f'(0) = 0$, $f''(0) = -1$.

$\Rightarrow f(0{,}02) \approx \underbrace{1 + 0 \cdot 0{,}02 - 1 \cdot \frac{(0{,}02)^2}{2}}_{0{,}9998}$

5. $$c_0 = \frac{1}{3}\underbrace{\int_0^3 e^t\,dt}_{[e^t]_0^3} = \frac{e^3-1}{3}$$

$$c_4 = \frac{1}{3}\int_0^3 \underbrace{e^{-2\pi i 4\frac{t}{3}} e^t}_{e^{(1-\frac{8}{3}\pi i)t}}\,dt$$

$$= \frac{1}{3}\left[\frac{e^{(1-\frac{8}{3}\pi i)t}}{1-\frac{8}{3}\pi i}\right]_0^3 = \frac{e^{3-8\pi i}-e^0}{3-8\pi i}$$

$$= \frac{e^3-1}{3-8\pi i}$$

6. $$\frac{\partial f}{\partial x} = 3x^2+6xy-6x+3y^2-8y+4 \overset{!}{=} 0$$

$$\frac{\partial f}{\partial y} = 3x^2+6xy-8x+3y^2-8y+4 \overset{!}{=} 0$$

Differenz davon: $2x = 0 \Longleftrightarrow x = 0$

$$0 = 3y^2-8y+4 \Longleftrightarrow y^2-\frac{8}{3}y+\frac{4}{3}=0$$

$$\Longleftrightarrow y = \frac{4}{3}\pm\sqrt{\underbrace{\frac{16}{9}-\frac{4}{3}}_{\frac{4}{9}}} = \frac{4}{3}\pm\frac{2}{3}$$

$$\Longleftrightarrow y = \frac{2}{3} \vee y = 2$$

$\downarrow$

Also lok. Extrema allenfalls an $(0|\frac{2}{3})$ oder $(0|2)$.

$$\frac{\partial^2 f}{\partial x^2} = 6x + 6y - 6, \quad \frac{\partial^2 f}{\partial y^2} = 6x + 6y - 8,$$
$$\frac{\partial^2 f}{\partial x \partial y} = 6x + 6y - 8.$$

Hesse-Matrix an $(0|\frac{2}{3})$: $\begin{pmatrix} -2 & -4 \\ -4 & -4 \end{pmatrix}$

det(%) < 0, also kein lok. Extr.

Hesse-Matrix an $(0|2)$: $\begin{pmatrix} 6 & 4 \\ 4 & 4 \end{pmatrix}$

det(%) > 0, $6 > 0$, also lok. Min.

7. $E_1: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix}$

$E_2: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} + \% + \%$

8. Die Matrix muss $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ und auch $\begin{pmatrix} 0 \\ 4 \\ 5 \end{pmatrix}$ zu $\vec{0}$ machen, darf aber die Richtung senkrecht zu beiden nicht zu $\vec{0}$ machen.

$$\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \times \begin{pmatrix} 0 \\ 4 \\ 5 \end{pmatrix} = \begin{pmatrix} -2 \\ -5 \\ 4 \end{pmatrix}$$

Nehme z.B. $\begin{pmatrix} -2 & -5 & 4 \\ -2 & -5 & 4 \\ -2 & -5 & 4 \end{pmatrix}$.

9. Ansatz: $y(x) = Ae^{5x}$

$$\Rightarrow A \cdot 25e^{5x} + 25 \cdot Ae^{5x} \overset{!}{=} e^{5x}$$

$$\Rightarrow A = 1/50$$

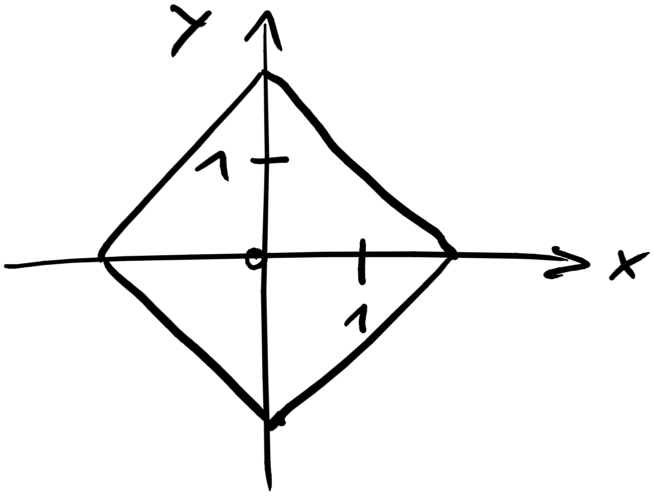
10. $-\dot{y}(0) - sy(0) + s^2 Y(s)$

$$-4y(0) + 4sY(s) + 5Y(s) = \frac{1}{s^2}$$

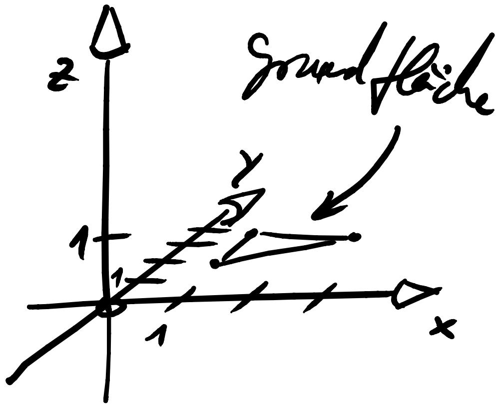
$$\Rightarrow Y(s) = \frac{\frac{1}{s^2} + 2 + s + 4}{s^2+4s+5} = \frac{\frac{1}{s^2}+s+6}{s^2+4s+5}$$

11. $3 - |x| - |y| = 1 \Leftrightarrow |x| + |y| = 2$

$$\Leftrightarrow y = \pm(2 - |x|)$$



12. $$V = \int_2^4 \underbrace{\left(\int_1^{y-1} 2x\,dx\right)}_{\underbrace{\left[x^2\right]_{x=1}^{x=y-1}}_{(y-1)^2-1^2}} dy$$



$$= \left[\frac{(y-1)^3}{3} - y\right]_2^4 = 9 - 4 - \frac{1}{3} + 2$$

$$= 6\,\frac{2}{3}$$