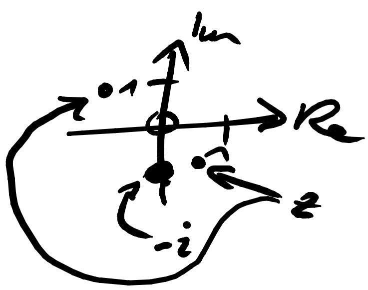
Mathematik I, 2017-07-27
Musterlösungen

1. $\log_{10}\left(\sqrt{x^2+1}\right) = 3 \iff \sqrt{x^2+1} = 10^3$

$\iff x^2+1 = 10^6 \iff x = \pm\sqrt{999999}$

2. $\underbrace{z^3 + iz}_{z\cdot(z^2+i)} = 0 \iff z = 0 \vee z^2 = -i$

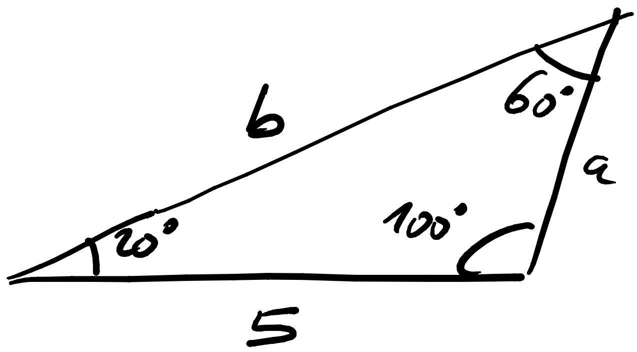


$\iff z = 0 + 0i$

$\vee\ z = \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i$

$\vee\ z = -\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$

3.



Eindeutig!

$a = \frac{\sin(20°)}{\sin(60°)} \cdot 5$

$b = \frac{\sin(100°)}{\sin(60°)} \cdot 5$

4. $\dfrac{x}{x^2-9}$ hat Polstellen 1. Ordnung bei $x = \pm 3$.

$x^2-9 = (x-3)(x+3)$

Zählergrad < Nennergrad ✓

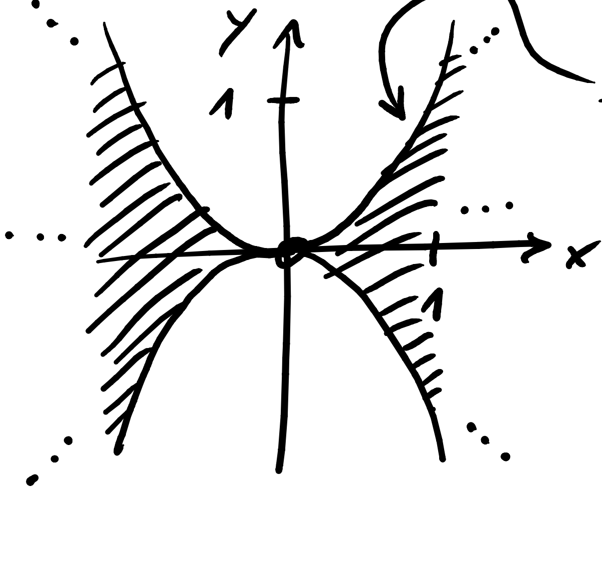
Ansatz: $\dfrac{x}{x^2-9} = \dfrac{A}{x-3} + \dfrac{B}{x+3} = \dfrac{A(x+3)+B(x-3)}{x^2-9}$

$$\Longleftrightarrow \begin{cases} A+B=1 \\ 3A-3B=0 \end{cases} \Longleftrightarrow A = \tfrac{1}{2} = B$$

5. $$\frac{d\,\frac{\sin(3x+1)}{\sqrt{x}}}{dx} = \frac{\cos(3x+1)\cdot 3\cdot\sqrt{x} - \sin(3x+1)\cdot\frac{1}{2\sqrt{x}}}{x}$$

6. $$P = \frac{5}{100}\cdot\frac{4}{99}\cdot\frac{3}{98}$$

7.



einschließlich Rand

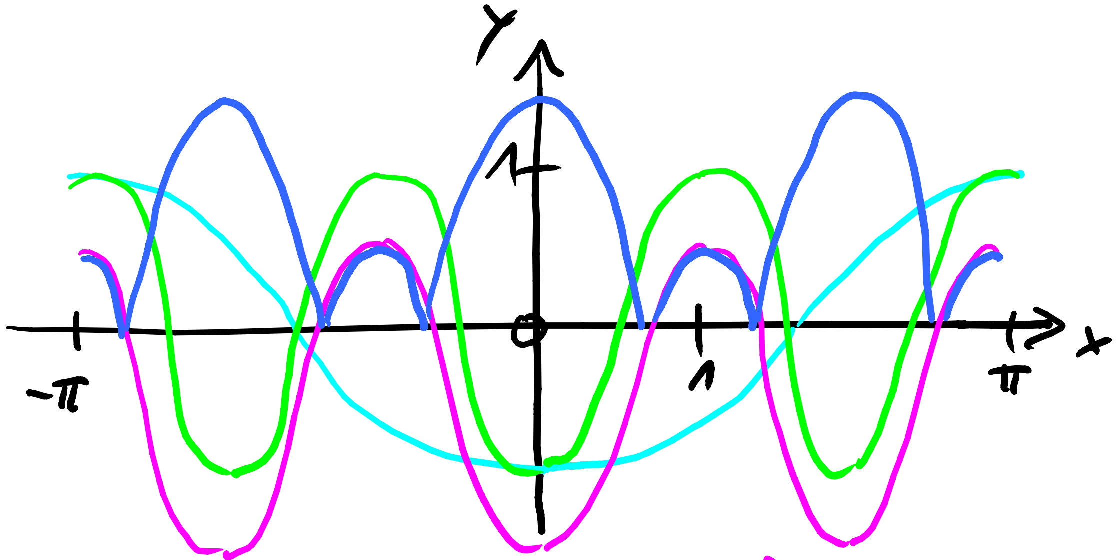
8. zum Beispiel:

$$p(x) = \overset{?}{c} \cdot (x+1)^2 (x-2)^2$$

$$3 \overset{!}{=} p(0) = c \cdot 1^2 \cdot (-2)^2 = 4c,$$

also wähle $c = \frac{3}{4}$.

9.



$\cos(x-\pi)$ $\qquad$ $\cos(3x-\pi)-\frac{1}{2}$

$\cos(3x-\pi)$ $\qquad$ $\left|\cos(3x-\pi)-\frac{1}{2}\right|$

10.

$$\frac{\sqrt{n^4+1}+\sin(n^6)}{3e^{5-n}+4n^2} = \frac{\overset{\to 1}{\sqrt{1+\frac{1}{n^4}}} + \overset{\to 0}{\frac{\sin(n^6)}{n^4}}}{\underbrace{\underbrace{\frac{3e^{5-n}}{n^2}}_{\to 0} + 4}_{\to 4}} \to \frac{1}{4}$$

11. $\int_0^1 \sin(x)\sqrt{\cos(x)}\,dx$

$u = \cos(x)$
$\frac{du}{dx} = -\sin(x)$

$= -\int_0^1 \frac{du}{dx}\sqrt{u(x)}\,dx$

$= -\int_{\cos(0)}^{\cos(1)} \sqrt{u}\,du = -\left[\frac{2}{3}u^{3/2}\right]_{\cos(0)}^{\cos(1)}$

$= \frac{2}{3}\left(1 - (\cos(1))^{3/2}\right)$

12. $E\left[(X+Y)^2\right] = E[X^2] + 2\underbrace{E[X\cdot Y]}_{=E[X]\cdot E[Y],}$
$+ E[Y^2]$

weil unabhängig

$= \underbrace{\int_0^1 x^2 \cdot 1\,dx}_{\left[\frac{x^3}{3}\right]_0^1 = \frac{1}{3}} + \cancel{2} \cdot \cancel{\frac{1}{2}} \cdot 3\frac{1}{2} + \frac{1}{2}\cdot 3^2 + \frac{1}{2}\cdot 4^2$

$= \frac{1}{3} + 3\frac{1}{2} + 4\frac{1}{2} + 8 = 16\frac{1}{3}$