Mathematik 2

2016-09-29

Musterlösungen

1.

$$\begin{matrix} 1 & 2 & -1 & 3 \\ 1 & 1 & 1 & 1 \\ 2 & 3 & 1 & 2 \end{matrix} \quad \begin{matrix} \\ \cdot(-1) \\ \cdot(-2) \end{matrix}$$

$$\begin{matrix} 1 & 2 & -1 & 3 \\ 0 & -1 & 2 & -2 \\ 0 & -1 & 3 & -4 \end{matrix} \quad \begin{matrix} \\ \\ \cdot(-1) \end{matrix}$$

$$\begin{matrix} 1 & 2 & -1 & 3 \\ 0 & -1 & 2 & -2 \\ 0 & 0 & 1 & -2 \end{matrix}$$

$\Rightarrow z = -2$

$y = -2$

$x = 3 + 2 \cdot 2 - 1 \cdot 2$
$= 5$

2. Homogene Form: $y'' + y' = 0$

Ansatz: $y(x) = e^{\lambda x}$

$\Rightarrow \lambda^2 + \lambda = 0 \Rightarrow \lambda = -1 \vee \lambda = 0$

$\Rightarrow$ allg. Lsg. der hom. Form: $y(x) = Ae^{-x} + B$

$\downarrow$

Suche eine spez. Lsg. der inh. Form.

Ansatz: $y(x) = Cx^2 + Dx + E$

$$\Rightarrow 2C + 2Cx + D = 4x$$

$$\Rightarrow C = 2 \wedge D = -4, \quad E \text{ ist beliebig.}$$

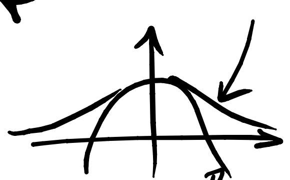
Damit allg. Lsg der urspgl. DGL:

$$y(x) = Ae^{-x} + B + 2x^2 - 4x$$

3. $$\frac{y'}{y} = -e^x \Rightarrow \underbrace{\int_3^{y_1} \frac{dy}{y}}_{\ln\left|\frac{y_1}{3}\right|} = \underbrace{-\int_5^{x_1} e^x dx}_{-e^{x_1}+e^5}$$

$$\Rightarrow y = 3e^{(e^5-e^x)}$$

4. $f(x) = e^{-x^2}$, Entwickeln an $x=0$



Schmiegeparabel

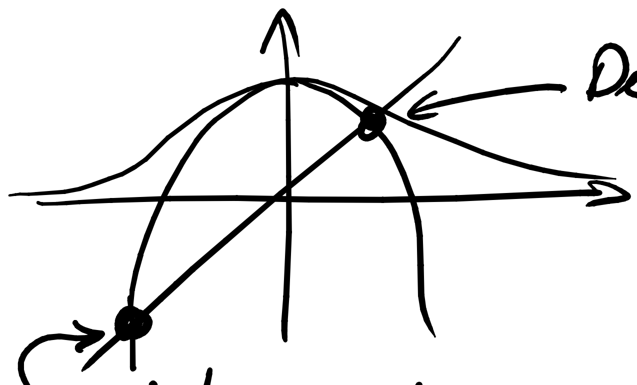
$$f'(x) = e^{-x^2} \cdot (-2x)$$
$$f''(x) = e^{-x^2} \cdot (-2x)^2 + e^{-x^2} \cdot (-2)$$
$$f(0) = 1, \quad f'(0) = 0, \quad f''(0) = -2$$

Also $f(x) \approx 1 - \frac{2}{2}x^2 = 1 - x^2$.

Zu lösen: $1 - x^2 \approx x$

$$\Longrightarrow x^2 + x - 1 \approx 0$$

$$\Longrightarrow x \approx \underbrace{-\frac{1}{2} \pm \sqrt{\frac{1}{4}+1}}_{\frac{-1\pm\sqrt{5}}{2}}$$



Das mit + ist der Interessante!

uninteressant

5. $$a_3 = \frac{\cancel{2}^{\,1}}{\cancel{6}_{\,3}} \int_{-2}^{4} \cos\left(\cancel{2}\pi \cdot \cancel{3}\frac{t}{\cancel{6}}\right)(1+t)\,dt$$

$$= \underbrace{\int_{-2}^{4} \cos(\pi t)\,dt}_{\underbrace{\left[\frac{1}{\pi}\sin(\pi t)\right]_{-2}^{4}}_{0}} + \underbrace{\int_{-2}^{4} \cos(\pi t)\, t\,dt}_{\underbrace{\left[\frac{1}{\pi}\sin(\pi t)\cdot t\right]_{-2}^{4}}_{0} - \int_{-2}^{4}\frac{1}{\pi}\sin(\pi t)\, 1\,dt}$$

$\uparrow$ $\frac{1}{\pi}\sin(\pi t)$ $\quad$ $\downarrow$ 1

$$= -\frac{1}{\pi}\left[-\frac{1}{\pi}\cos(\pi t)\right]_{-2}^{4} = 0.$$

6. $\frac{\partial f}{\partial x} = 6x^2 + 2x + 6y^2 + 6y$

$\frac{\partial f}{\partial y} = 12xy + 6x + 2y$

Wird beides $= 0$ an $\left(\frac{1}{3} \middle| -\frac{1}{3}\right)$, also horiz. Tangentialebene.

an $\left(\frac{1}{3} \middle| -\frac{1}{3}\right)$:

$\frac{\partial^2 f}{\partial x^2} = 12x + 2 \longrightarrow 6$

$\frac{\partial^2 f}{\partial y^2} = 12x + 2 \longrightarrow 6$

$\frac{\partial^2 f}{\partial x \partial y} = 12y + 6 \longrightarrow 2$

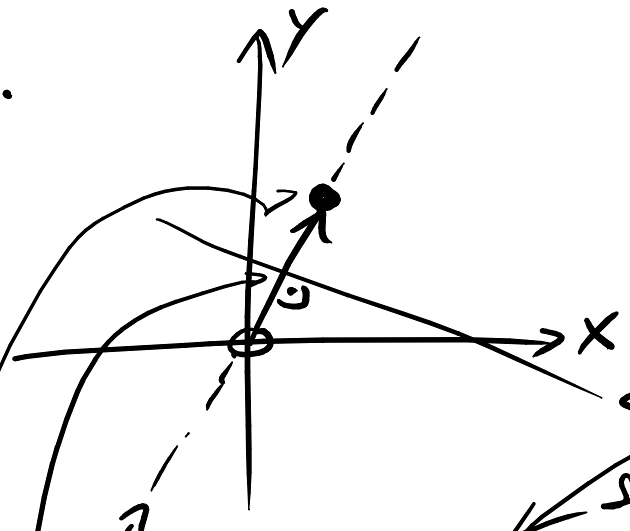
Also Hesse-Matrix an $\left(\frac{1}{3} \middle| -\frac{1}{3}\right)$:

$$\begin{pmatrix} 6 & 2 \\ 2 & 6 \end{pmatrix}.$$

Determinante davon $= 6^2 - 2^2 > 0$, also lok. Max. oder lok. Min.

positiv, also lok. Min.

7.



$$\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}1\\2\end{pmatrix} + \lambda\begin{pmatrix}3\\-1\end{pmatrix}$$

senkrecht

$$\begin{pmatrix}x\\y\end{pmatrix} = \mu\begin{pmatrix}1\\3\end{pmatrix}$$

Schnittpunkt:

$$\begin{pmatrix}1\\2\end{pmatrix} + \lambda\begin{pmatrix}3\\-1\end{pmatrix} = \mu\begin{pmatrix}1\\3\end{pmatrix}$$

$$\Longleftrightarrow \lambda\begin{pmatrix}3\\-1\end{pmatrix} + \mu\begin{pmatrix}-1\\-3\end{pmatrix} = \begin{pmatrix}-1\\-2\end{pmatrix}$$

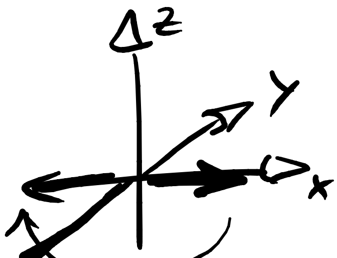
$$\Longrightarrow \mu = \frac{\begin{vmatrix}3&-1\\-1&-2\end{vmatrix}}{\begin{vmatrix}3&-1\\-1&-3\end{vmatrix}} = \frac{-6-1}{-9-1} = \frac{7}{10}$$

Also Schnittpunkt $= \frac{7}{10}\cdot\begin{pmatrix}1\\3\end{pmatrix}$.

Also Spiegelbild $= 2\cdot\% = \begin{pmatrix}7/5\\21/5\end{pmatrix}$.

8. EW 1, EV z.B. $\begin{pmatrix}0\\1\\0\end{pmatrix}$ ((Rotationsachse!))

EW −1, EV z.B. $\begin{pmatrix}1\\0\\0\end{pmatrix}$



9. Nein, weil sie mindestens den Defekt 2 hat:

$$4 = \text{Rang} + \text{Defekt}$$

≤ 2, weil Bild im $\mathbb{R}^2$

10. Ansatz: $x(t) = A\cos(3t) + B\sin(3t)$

$\Longrightarrow \ddot{x}(t) = -9A\cos(3t) - 9B\sin(3t)$

$\Longrightarrow \ddot{x} + 9x = 0$ ↯

neuer Ansatz: $x(t) = At\cos(3t)$

$\Longrightarrow \dot{x}(t) = A\cos(3t) - 3At\sin(3t)$

$\Longrightarrow \ddot{x}(t) = -6A\sin(3t) - 9At\cos(3t)$

Einsetzen:

$\ddot{x} + 9x = -6A\sin(3t) \cancel{- 9At\cos(3t) + 9At\cos(3t)}$

$\Longrightarrow A = -\frac{1}{6}$ und DGL gelöst.

11. $s^3+s^2 = s^2(s+1)$.

Partialbruchzerlegung:

$$\frac{s+3}{s^3+s^2} = \frac{A}{s^2} + \frac{B}{s} + \frac{C}{s+1}$$

$$= \frac{A(s+1) + Bs(s+1) + Cs^2}{s^3+s^2}$$

$\Longrightarrow$ s^2: $B+C=0$ $\longrightarrow$ $C=2$

s^1: $A+B=1$ $\longrightarrow$ $B=-2$

s^0: $A=3$

$$\Longrightarrow \frac{s+3}{s^3+s^2} = \frac{3}{s^2} - \frac{2}{s} + \frac{2}{s+1}$$

$$\Longrightarrow y(t) = 3t - 2 + 2e^{-t}$$

12. $f(x_0,y_0) = \frac{4}{4}e^0 = 1$

Höhenlinie: $\frac{4}{y^2}e^{x-1} = 1 \iff y^2 = 4e^{x-1}$

$$\iff y = \pm 2e^{(x-1)/2}$$