Mathematik 2

2016-07-15

Musterlösungen

1) Ein Richtungsvektor von g ist z.B. $\begin{pmatrix}1\\4\end{pmatrix}$; einer von h ist z.B. $\begin{pmatrix}5\\-5\end{pmatrix}$.

$$\begin{pmatrix}1\\4\end{pmatrix}\cdot\begin{pmatrix}5\\-5\end{pmatrix} = -15 < 0,$$

also so mehr als $90°$.

Nehme also $\begin{pmatrix}-5\\5\end{pmatrix}$ statt $\begin{pmatrix}5\\-5\end{pmatrix}$.

$$\text{Winkel} = \operatorname{acos}\left(\frac{15}{\left\|\begin{pmatrix}1\\4\end{pmatrix}\right\|\cdot\left\|\begin{pmatrix}-5\\5\end{pmatrix}\right\|}\right)$$

$$= \operatorname{acos}\left(\frac{15}{\sqrt{17\cdot 50}}\right)$$

2)

$$\begin{pmatrix}0 & 1 & 2 & 1\\ 1 & 2 & 4 & 3\\ 2 & 0 & 0 & 2\end{pmatrix}$$

$\cdot 2$

$+$

Also Rang = 2. Gleichungssysteme mit dieser K.-M. sind <u>nicht</u> immer lösbar.

3) $$y' = x^2 y$$

$$\Rightarrow \frac{dy}{y} = x^2 dx$$

$$\Rightarrow \underbrace{\int_5^{y_1} \frac{dy}{y}}_{\ln|y_1| - \ln|5|} = \underbrace{\int_4^{x_1} x^2 dx}_{\frac{1}{3}(x_1^3 - 4^3)}$$

$$\Rightarrow y_1 = 5 e^{\frac{1}{3}(x_1^3 - 4^3)}$$

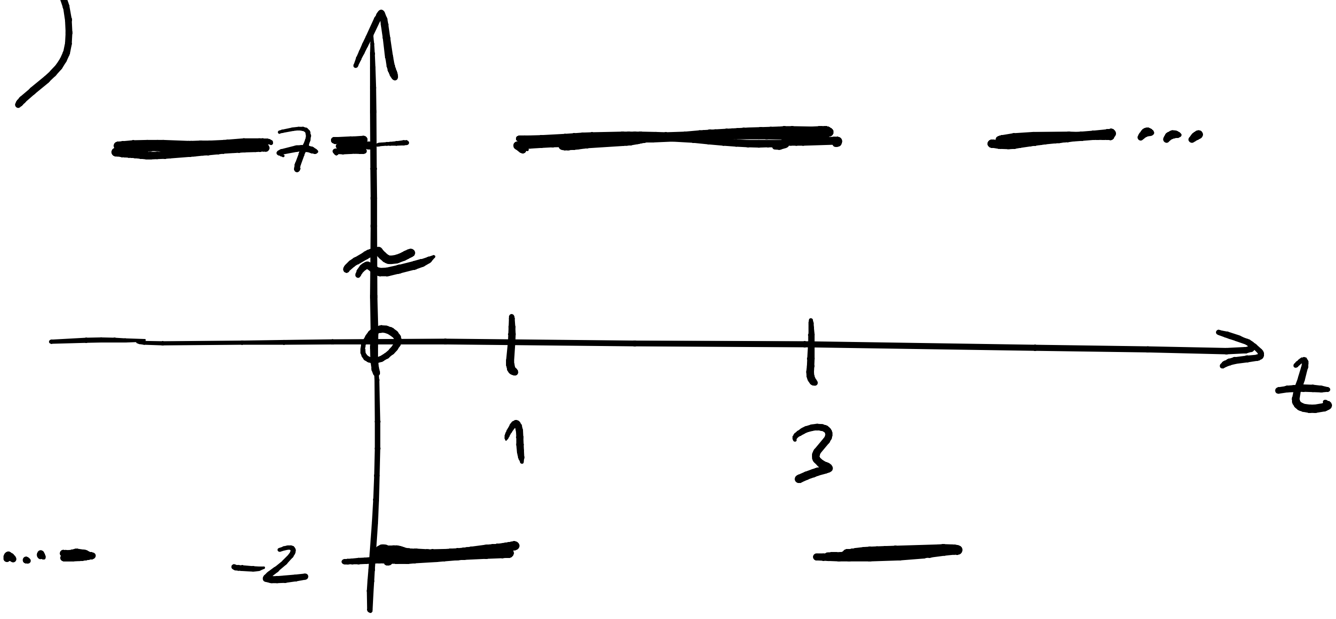
4) $$f(x) = \frac{1}{x^5} = x^{-5}, \quad x_0 = 2, \quad h = -0{,}02$$

$$f'(x) = -5x^{-6}, \quad f''(x) = 30x^{-7}.$$

$$f(x_0) = \frac{1}{32}, \quad f'(x_0) = \frac{-5}{64}, \quad f''(x_0) = \frac{\cancel{30}\ 15}{\cancel{128}\ 64}$$

$$\Rightarrow \frac{1}{1{,}98^5} \approx \frac{1}{32} + \frac{5}{64} \cdot 0{,}02 + \frac{15}{64} \frac{0{,}02^2}{2}$$

5)



$$c_0 = \frac{1}{3}\int_0^3 f(t) = \frac{1}{3}\left(-2\cdot 1 + 7\cdot 2\right) = 4$$

$$c_5 = \frac{1}{3}\int_0^3 e^{-2\pi i\cdot 5\frac{t}{3}} f(t)\,dt$$

$$= \frac{1}{3}\left(-2\cdot\underbrace{\int_0^1 e^{-2\pi i\cdot 5\frac{t}{3}}\,dt}_{\frac{e^{-2\pi i\cdot 5\cdot\frac{1}{3}}-1}{-2\pi i\cdot\frac{5}{3}}} + 7\cdot\underbrace{\int_1^3 e^{-2\pi i\cdot 5\frac{t}{3}}\,dt}_{\frac{e^{-2\pi i\cdot 5\cdot 1}-e^{-2\pi i\cdot 5\frac{1}{3}}}{-2\pi i\cdot\frac{5}{3}}}\right)$$

$$= \frac{1}{\cancel{3}}\,\frac{i}{2\pi\cdot\frac{5}{\cancel{3}}}\left(-2e^{-2\pi i\frac{5}{3}} + 2 + 7\underbrace{e^{-10\pi i}}_{1} - 7e^{-2\pi i\frac{5}{3}}\right)$$

$$= \frac{9i}{10\pi}\left(1 - e^{-\frac{10}{3}\pi i}\right)$$

6) $$\frac{\partial f}{\partial x} = \sin(x-y) + (x+y)\cos(x-y)$$

$$\frac{\partial f}{\partial y} = \sin(x-y) - (x+y)\cos(x-y)$$

$$\Rightarrow \frac{\partial f}{\partial x}(\pi,-\pi) = \underbrace{\sin(2\pi)}_{0} + 0 \cdot \ldots = 0$$

$$\frac{\partial f}{\partial y}(\pi,-\pi) = \ldots - 0 \cdot \ldots = 0$$

Könnte also ein lok. Min. sein!

$$\frac{\partial^2 f}{\partial x^2} = \cos(x-y) + 1 \cdot \cos(x-y)$$
$$- (x+y)\sin(x-y)$$
$$= 2\cos(x-y) - (x+y)\sin(x-y)$$

$$\frac{\partial^2 f}{\partial y^2} = -2\cos(x-y) - (x+y)\sin(x-y)$$

$(-\sin)\cdot(-1)$

$$\frac{\partial^2 f}{\partial x \partial y} = \cos(x-y) - \cos(x-y) + (x+y)\sin(x-y)$$
$$= (x+y)\sin(x-y)$$

$$\Rightarrow \frac{\partial^2 f}{\partial x^2}(\pi,-\pi) = 2 \cdot 1 - 0 \cdot \ldots = 2$$

$$\frac{\partial^2 f}{\partial y^2}(\pi,-\pi) = -2 \cdot 1 - 0 \cdot \ldots = -2$$

$$\frac{\partial^2 f}{\partial x \partial y}(\pi,-\pi) = 0$$

Also Hesse-Matrix $= \begin{pmatrix} 2 & 0 \\ 0 & -2 \end{pmatrix}$.

det(%) $= -4 < 0$.

Also Sattel, kein lok. Min.

7) z.B. $\begin{vmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \boxed{12} & \boxed{0} & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{vmatrix}$

8) $2x + 3y = 0 \iff \begin{pmatrix} 2 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = 0$

d.h. $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ ist senkrecht zur Gerade, an der gespiegelt wird.

- E.W. 1, E.V. dazu z.B. $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$.
- E.W. -1, E.V. dazu z.B. $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$.

((senkrecht zu $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$!))

9) $y'' - 3y' + x = 0$ linear, inhomogen

$\downarrow$

Schritt 1: allg. Lsg. der homogenen Form

$$y'' - 3y' = 0$$

Ansatz: $y(x) = e^{\lambda x}$

$$\Rightarrow \lambda^2 - 3\lambda = 0$$

$$\Rightarrow \lambda = 0 \vee \lambda = 3$$

Allg. Lsg. der homogenen Form ist also: $y(x) = A\underbrace{e^{0x}}_{1} + Be^{3x}$

Schritt 2: eine spez. Lsg. der inhomog. Form

$$y'' - 3y' = -x$$

Ansatz: $y(x) = Cx^2 + Dx$

$$\Rightarrow 2C - 3(2Cx + D) = -x$$

$$\Rightarrow C = \frac{1}{6},\ D = \frac{2}{3}C = \frac{1}{9}$$

Schritt 3: allg. Lsg. der inh. Form:

$$y(x) = A + Be^{3x} + \frac{x^2}{6} + \frac{x}{9}$$

Schritt 4: Anfangsbedingung:

$$2 \stackrel{!}{=} y(0) = A + B\underbrace{\cancel{e^{3\cdot 0}}}^{1} + 0$$

$$1 \stackrel{!}{=} y'(0) = 3B\underbrace{\cancel{e^{3\cdot 0}}}_{1} + 0 + \frac{1}{9}$$

$$\Rightarrow \quad B = \frac{1}{3}\left(1-\frac{1}{9}\right) = \frac{8}{27}$$

$$A = 2 - B = \frac{46}{27}$$

10) $\ddot{x} + 8\dot{x} + 16x = 0$

linear, homogen

Ansatz: $x(t) = e^{\lambda t}$

$$\Rightarrow \lambda^2 + 8\lambda + 16 = 0$$

$$\Rightarrow \lambda = -\frac{8}{2} \pm \sqrt{16-16} = -4$$

Nur ein λ!

Also allg. Lsg.: $x(t) = Ae^{-4t} + Bte^{-4t}$.

!

11) $$\frac{1}{s^4-1} = \frac{1}{(s^2+1)\underbrace{(s^2-1)}_{(s+1)(s-1)}}$$

$$= \frac{A+Bs}{s^2+1} + \frac{C}{s+1} + \frac{D}{s-1}$$

$$= \frac{(A+Bs)(s+1)(s-1) + C(s^2+1)(s-1) + D(s^2+1)(s+1)}{(s^2+1)(s+1)(s-1)}$$

$$\Longrightarrow 1 = (A + Bs)(s^2-1) + C(s^2+1)(s-1) + D(s^2+1)(s+1)$$

$$\Longrightarrow \begin{array}{lll} s^0 & 1 = -A - C + D & \mathrm{I} \\ s^1 & 0 = -B + C + D & \mathrm{II} \\ s^2 & 0 = A - C + D & \mathrm{III} \\ s^3 & 0 = B + C + D & \mathrm{IV} \end{array}$$

II & IV $\Longrightarrow B = 0, \; C = -D$

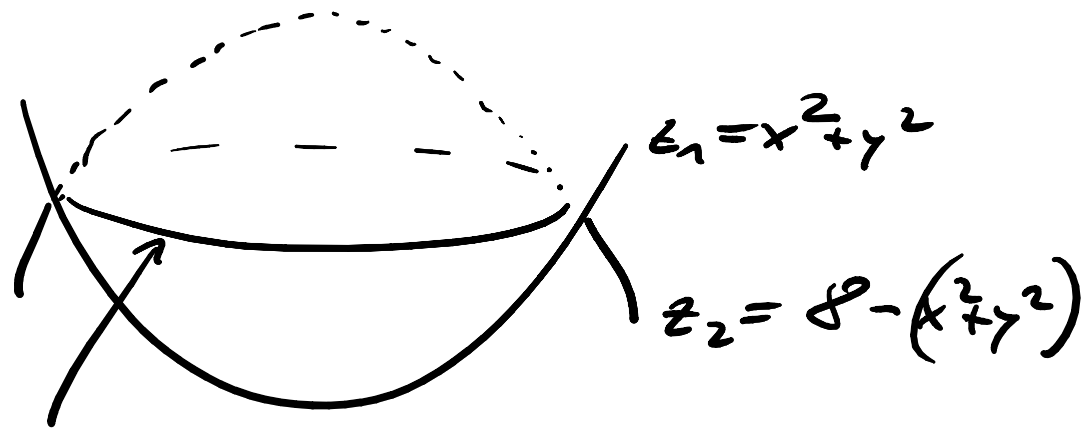
I & III $\Longrightarrow A = -\frac{1}{2}$

$\mathrm{III} \Longrightarrow C = -\frac{1}{4}, \; D = \frac{1}{4}$

$$\Rightarrow Y(s) = \frac{1}{s^4-1} = -\frac{1}{2}\frac{1}{s^2+1} - \frac{1}{4}\frac{1}{s+1} + \frac{1}{4}\frac{1}{s-1}.$$

Also $y(t) = -\frac{1}{2}\sin(t) - \frac{1}{4}e^{-t} + \frac{1}{4}e^{t}$.

12)



$z_1 = z_2$,

also $x^2+y^2 = 8-(x^2+y^2)$, also $x^2+y^2 = 4$

$\Rightarrow$ Integrationsgebiet: Kreisscheibe um 0 mit Radius 2

$$\text{Volumen} = \iint \Big(\underbrace{\big(8-(x^2+y^2)\big)-\big(x^2+y^2\big)}_{8\,-\,2(x^2+y^2)} \Big)\, dx\, dy$$

$$= \int_0^{2\pi} \Bigg(\underbrace{\int_0^2 \left(8-2r^2\right) r\,dr}_{\left[\cancel{8}^{\,4}\frac{r^2}{\cancel{2}} - \cancel{2}\frac{r^4}{\cancel{4}_{\,2}}\right]_0^2} \Bigg)\, d\varphi$$

$$= 2\pi \cdot \left(4 \cdot 4 - 8\right) = 16\pi.$$