Mathematik 2
2015-07-17
Musterlösungen

1.

$$\begin{array}{cccc} 1 & 1 & -1 & 1 \\ 2 & 1 & 0 & 2 \\ 4 & 0 & 3 & 3 \end{array} \quad \cdot(-2) \quad \cdot(-4)$$

$$\begin{array}{cccc} 1 & 1 & -1 & 1 \\ 0 & -1 & 2 & 0 \\ 0 & -4 & 7 & -1 \end{array} \quad \cdot(-4)$$

$$\begin{array}{cccc} 1 & 1 & -1 & 1 \\ 0 & -1 & 2 & 0 \\ 0 & 0 & -1 & -1 \end{array}$$

$$\Rightarrow z = 1$$

$$y = 2$$

$$x = 0$$

2. $\begin{vmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 & 2 \\ 0 & 0 & 0 & 3 & 0 \\ 2 & 0 & 0 & 0 & 0 \\ 0 & 3 & 2 & 0 & 1 \end{vmatrix} = -2 \cdot \begin{vmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 2 \\ 0 & 0 & 3 & 0 \\ 3 & 2 & 0 & 1 \end{vmatrix}$

$$= 2 \cdot 1 \cdot \underbrace{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 3 & 0 \\ 2 & 0 & 1 \end{vmatrix}}_{\substack{1\cdot 3\cdot 1 + 0 + 0 \\ -2\cdot 3\cdot 2 - 0 - 0}} = -18$$

3. <u>Allg. Lsg. der homogenen Form:</u>

$\ddot{x} - x = 0$

Ansatz: $x(t) = e^{\lambda t}$

$\Rightarrow \lambda^2 - 1 = 0 \Rightarrow \lambda = \pm 1$

Also $x(t) = Ae^{t} + Be^{-t}$

<u>Spez. Lsg. der inhomogenen Form:</u>

Offensichtlich $x(t) = -t$

Also allg. Lsg.:

$$x(t) = Ae^{t} + Be^{-t} - t$$

Anfangsbedingung:

$$\begin{cases} 3 \overset{!}{=} Ae^{0} + Be^{-0} - 0 \\ 5 \overset{!}{=} Ae^{0} - Be^{-0} - 1 \end{cases}$$

$$\Longrightarrow \begin{cases} 3+5 = 2A - 1 \\ 3-5 = 2B + 1 \end{cases}$$

$$\Longrightarrow A = \frac{9}{2}, \quad B = -\frac{3}{2}$$

Also $x(t) = \frac{9}{2}e^{t} - \frac{3}{2}e^{-t} - t$.

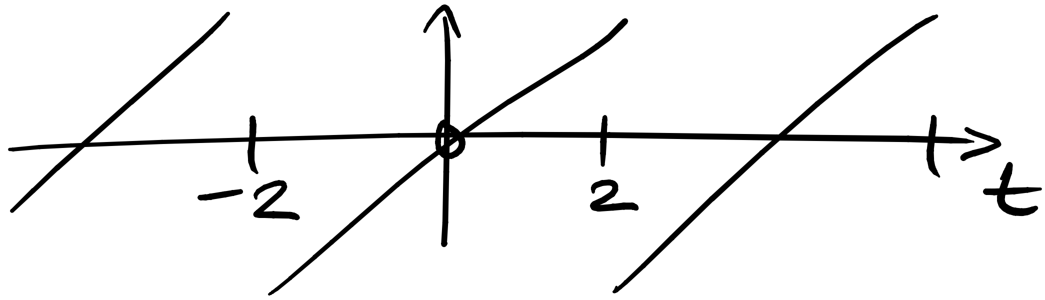
4. $y' = xy$

Trennung der Variablen:

$$\frac{dy}{y} = x\,dx \Longrightarrow \underbrace{\int_2^{y_1} \frac{dy}{y}}_{\ln|y_1/2|} = \underbrace{\int_3^{x_1} x\,dx}_{\frac{1}{2}(x_1^2 - 9)}$$

$$\Longrightarrow y_1 = 2\exp\left(\tfrac{1}{2}(x_1^2 - 9)\right)$$

5.



ungerade Funktion $\Rightarrow a_{\dots} = 0$

$$b_5 = \frac{\cancel{2}^{\,1}}{\cancel{4}_{\,2}} \int_{-2}^{2} \sin\left(\cancel{2}\pi 5 \frac{t}{\cancel{4}_{\,2}}\right) t \, dt$$

$$\uparrow \qquad \downarrow$$

$$\frac{-\cos\left(\frac{5}{2}\pi t\right)}{\frac{5}{2}\pi} \qquad 1$$

$$= \frac{1}{2}\left(\left[\frac{-\cos\left(\frac{5}{2}\pi t\right)}{\frac{5}{2}\pi} t \right]_{-2}^{2} + \underbrace{\int_{-2}^{2} \frac{\cos\left(\frac{5}{2}\pi t\right)}{\frac{5}{2}\pi} dt}_{\left[\frac{\sin\left(\frac{5}{2}\pi t\right)}{\left(\frac{5}{2}\pi\right)^2} \right]_{-2}^{2}} \right)$$

$$= \frac{1}{2}\left(\frac{-\cos(5\pi)\cdot 2 + \cos(-5\pi)\cdot(-2)}{\frac{5}{2}\pi} + \frac{\sin(5\pi) + \sin(5\pi)}{\left(\frac{5}{2}\pi\right)^2} \right)$$

$$= \frac{1}{\cancel{2}}\left(\frac{-2\cdot 2 \overbrace{\cos(5\pi)}^{-1}}{\frac{5}{\cancel{2}}\pi} + 0 \right) = \frac{4}{5\pi}$$

6. $f(x) = e^{\sin(x)}$

$$f'(x) = e^{\sin(x)} \cos(x)$$

$$f''(x) = e^{\sin(x)} \cos^2(x) - e^{\sin(x)} \sin(x)$$

$$= e^{\sin(x)} \left(\cos^2(x) - \sin(x)\right)$$

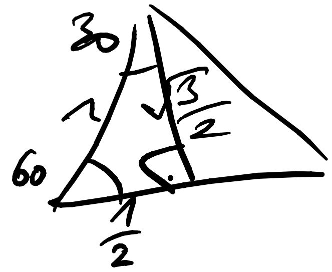
$$f(\pi) = e^{\sin(\pi)} = 1$$

$$f'(\pi) = 1 \cdot \cos(\pi) = -1$$

$$f''(\pi) = 1 \cdot \left((-1)^2 - 0\right) = 1$$

$$\Longrightarrow$$

$$f(\pi + 0{,}01) \approx \underbrace{1 - 1 \cdot 0{,}01 + 1 \cdot \frac{0{,}01^2}{2}}_{0{,}99005}$$



7. $g: \begin{pmatrix} 2 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 4 \end{pmatrix}$

Drehe Richtungsvektor:

$$\begin{pmatrix} \cos(30°) & -\sin(30°) \\ \sin(30°) & \cos(30°) \end{pmatrix} \begin{pmatrix} 1 \\ 4 \end{pmatrix}$$

$$= \begin{pmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{pmatrix} \begin{pmatrix} 1 \\ 4 \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{3}}{2} - 2 \\ \frac{1}{2} - 2\sqrt{3} \end{pmatrix}$$

Also nehme z.B. die Grade

$$\begin{pmatrix} 2 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} \frac{\sqrt{3}}{2} - 2 \\ \frac{1}{2} - 2\sqrt{3} \end{pmatrix}.$$

8. Bild $=$ Grade $\lambda \begin{pmatrix} 3 \\ 4 \end{pmatrix}$

Kern $=$ Grade $\lambda \begin{pmatrix} 2 \\ -1 \end{pmatrix}$

Rang $= 1$

Defekt $= 1$

9. Gradengleichung: $\begin{pmatrix} x \\ y \end{pmatrix} = \lambda \begin{pmatrix} 2 \\ -1 \end{pmatrix}$.

Gesuchte Matrix sei $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$.

$$\begin{cases} \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \end{pmatrix} & \text{(längs Achse)} \\ \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 1 \\ 2 \end{pmatrix} = -\begin{pmatrix} 1 \\ 2 \end{pmatrix} & \text{(quer zu Achse)} \end{cases}$$

$$\Longleftrightarrow \left\{ \begin{aligned} 2a - b &= 2 \\ 2c - d &= -1 \\ a + 2b &= -1 \\ c + 2d &= -2 \end{aligned} \right\} \Longleftrightarrow \left\{ \begin{aligned} a &= 3/5 \\ b &= -4/5 \\ c &= -4/5 \\ d &= -3/5 \end{aligned} \right.$$

Also Matrix $= \begin{pmatrix} \frac{3}{5} & -\frac{4}{5} \\ -\frac{4}{5} & -\frac{3}{5} \end{pmatrix}$.

10. $y''' - y = e^x$

Ansatz: $y = Ae^x$

$\Longrightarrow Ae^x - Ae^x = 0 \neq e^x$ ↯

neuer Ansatz: $y = Axe^x$

$\Longrightarrow y' = Ae^x + Axe^x$

$y'' = Ae^x + Ae^x + Axe^x$

$y''' = Ae^x + Ae^x + Ae^x + Axe^x$

$\Longrightarrow 3Ae^x + \cancel{Axe^x} - \cancel{Axe^x} = e^x$

$\Longrightarrow A = 1/3.$

11. $Y(s) = \dfrac{1}{s^4 - s^2} = \dfrac{1}{s^2(s^2-1)}$

$= \dfrac{1}{s^2(s+1)(s-1)}$

$= \dfrac{A}{s^2} + \dfrac{B}{s} + \dfrac{C}{s+1} + \dfrac{D}{s-1}$

mit $A = -1$, $C = -\frac{1}{2}$, $D = \frac{1}{2}$

„händewedelnd"

B bestimmen:

$$1 \overset{!}{=} A(s^2-1) + Bs(s^2-1) + Cs^2(s-1) + Ds^2(s+1)$$

z.B. Koeffizient von s^1 betrachten:

$$0 = -B$$

Also $y(t) = -t - \frac{1}{2}e^{-t} + \frac{1}{2}e^{t}$.

12. Zum Beispiel:

$$f(x,y) = (x-1)^2 - (y-2)^2$$