Mathematik 2

2015-02-03

Musterlösungen

1) Ebenengleichung: $\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}1\\2\\3\end{pmatrix} + \lambda\begin{pmatrix}1\\1\\1\end{pmatrix} + \mu\begin{pmatrix}2\\0\\0\end{pmatrix}$

Schnitt mit x-Achse?

$$\begin{pmatrix}x\\0\\0\end{pmatrix} = \begin{pmatrix}1\\2\\3\end{pmatrix} + \lambda\begin{pmatrix}1\\1\\1\end{pmatrix} + \mu\begin{pmatrix}2\\0\\0\end{pmatrix}$$

$$\Leftrightarrow \begin{cases} x = 1 + \lambda + 2\mu \\ 0 = 2 + \lambda \\ 0 = 3 + \lambda \end{cases} \quad \lightning$$

also leere Schnittmenge

2) $$0 \overset{!}{=} \begin{vmatrix} 5-\lambda & 4 & 3 \\ 0 & 2-\lambda & 0 \\ 0 & 1 & -\lambda \end{vmatrix}$$

$$= (5-\lambda)(2-\lambda)(-\lambda) + 0 + 0 - 0 - 0 - 0$$

$$\Leftrightarrow \lambda = 5 \vee \lambda = 2 \vee \lambda = 0$$

3) $$\frac{y'}{\cos(x)+1} \overset{!}{=} 7 \wedge y(2) \overset{!}{=} 3$$

$$\Leftrightarrow y(x_1) - 3 = \int_2^{x_1} 7(\cos(x)+1)\,dx = 7\Big[\sin(x) + x\Big]_2^{x_1}$$

$$\Leftrightarrow y(x_1) = 3 + 7(\sin(x_1) + x_1 - \sin(2) - 2)$$
$$= -11 + 7\sin(x_1) + 7x_1 - 7\sin(2)$$

4) $$y' - y \overset{!}{=} e^x \wedge y(0) \overset{!}{=} 5$$

Allg. Lösung für homogene Form: $y(x) = Ae^x$

Suche spez. Lösung für inhomogene Form.

Ansatz: $y(x) = Be^x \Rightarrow 0 = e^x$ $\lightning$

neuer Ansatz: $y(x) = Bxe^x \Rightarrow Be^x + \cancel{Bxe^x} - \cancel{Bxe^x} \overset{!}{=} e^x$

$\Rightarrow B = 1$

Allgemeine Lösung der inhomogenen Form:

$$y(x) = A e^{x} + x e^{x}$$

$$5 \overset{!}{=} y(0) = A e^{0} + 0 e^{0} \Rightarrow A = 5$$

5)

$$f(x) = \sqrt{\cos(x)} \qquad \Rightarrow f(0) = \sqrt{1} = 1$$

$$f'(x) = \frac{1}{2\sqrt{\cos(x)}} \cdot (-\sin(x)) \qquad \Rightarrow f'(0) = 0$$

$$f''(x) = -\frac{1}{4\sqrt{\cos(x)}^{3}} (-\sin(x))^{2} + \frac{1}{2\sqrt{\cos(x)}} \cdot (-\cos(x))$$

$$\Rightarrow f''(0) = 0 + \frac{1}{2 \cdot \sqrt{1}} \cdot (-1) = -\frac{1}{2}$$

Also $f(0{,}01) \approx \underbrace{1 + 0 \cdot 0{,}01 + \left(-\frac{1}{2}\right) \cdot \frac{0{,}01^{2}}{2}}_{0{,}999975}$

6)

$$f(x,y) = x^{2}y^{2} + 3x^{2} + y^{2} + 4$$

$$\frac{\partial f}{\partial x} = 2xy^{2} + 6x = x(2y^{2}+6)$$

$$\frac{\partial f}{\partial y} = 2x^{2}y + 2y = y(2x^{2}+2)$$

Werden nie null!

Notwendig: $\frac{\partial f}{\partial x} = 0 \wedge \frac{\partial f}{\partial y} = 0$

$\Rightarrow x = 0 = y$. Allenfalls hier kann lok. Extremum sein!

$$\frac{\partial^{2} f}{\partial x^{2}} = 2y^{2}+6, \quad \frac{\partial^{2} f}{\partial y^{2}} = 2x^{2}+2, \quad \frac{\partial^{2} f}{\partial x \partial y} = 4xy$$

$\Rightarrow$ Hesse-Matrix an $(0|0)$ ist $\begin{pmatrix} 6 & 0 \\ 0 & 2 \end{pmatrix}$.

Offensichtlich zwei positive Eigenwerte (nämlich 6 und 2), also hinreichend für lokales Minimum.

7) Gleichung der Geraden:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 3 \\ 0 \end{pmatrix}$$

Ebene $\perp$ dazu z.B.:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 7 \\ 13 \\ 42 \end{pmatrix} + \mu \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} + \nu \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$$

8) Ebene $x+y+z=0$ wird z.B. aufgespannt durch die Vektoren $\begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$ und $\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$.

Also Matrix z.B. $\begin{pmatrix} 1 & 1 & 42 \\ 1 & -1 & -42 \\ -2 & 0 & 0 \end{pmatrix}$.

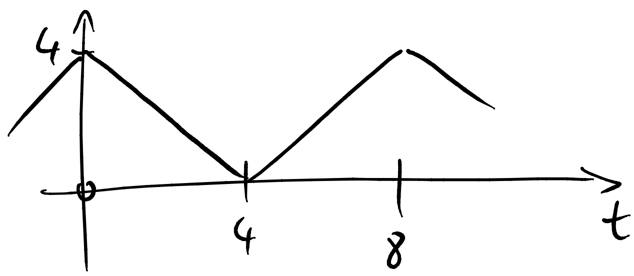
9) $y''' + y = x^3$

Ansatz: $y(x) = Ax^3 + Bx^2 + Cx + D$

$\Rightarrow 6A + Ax^3 + Bx^2 + Cx + D = x^3$

$\Rightarrow A = 1 \wedge B = 0 \wedge C = 0 \wedge D = -6$

10)



Gerade Funktion!

$\Downarrow$

Alle b sind 0.

$$a_3 = \underbrace{\frac{2}{8} \int_0^8 \cos\left(2\pi \cdot 3\frac{t}{8}\right) f(t)\, dt}_{2 \cdot \int_0^4 \cos\left(2\pi \cdot 3\frac{t}{8}\right)(4-t)\, dt}$$

$\uparrow$ $\frac{1}{2\pi \cdot 3/8} \sin\left(2\pi \cdot 3\frac{t}{8}\right)$ $\downarrow$ -1 $\downarrow$

$$= \frac{\cancel{4}^{1}}{\cancel{8}_{2}} \left(\left[\frac{1}{2\pi 3/8} \sin(2\pi \cdot 3t/8) \cdot (4-t) \right]_0^4 - \int_0^4 \frac{1}{2\pi 3/8} \sin(2\pi \cdot 3t/8) \cdot (-1)\, dt \right)$$

$$= \frac{1}{2} \left(0 + \frac{1}{\pi \cdot 3/4} \underbrace{\int_0^4 \sin(\pi \cdot 3t/4)\, dt}_{\left[\frac{1}{\pi \cdot 3/4} \left(-\cos\left(\pi \cdot \frac{3t}{4}\right)\right) \right]_0^4} \right)$$

$$= \frac{1}{2} \frac{1}{(\pi \cdot 3/4)^2} \left(-\underbrace{\cos(3\pi)}_{-1} - - \underbrace{\cos(0)}_{1} \right)$$

$$= \frac{1}{\cancel{2}} \frac{1}{(\pi \cdot \frac{3}{4})^2} \left(\cancel{1+1} \right) = \frac{16}{9\pi^2}$$

11) $$\frac{1}{s^4 + 9s^2} = \frac{1}{s^2(s^2+9)} = \frac{A}{s^2} + \frac{B}{s} + \frac{C+Ds}{s^2+9}$$

$$= \frac{As^2 + Bs^3 + 9A + 9Bs + Cs^2 + Ds^3}{s^2(s^2+9)}$$

$$\Rightarrow 1 = s^3(B+D) + s^2(A+C) + s \cdot 9B + 9A$$

$$\Rightarrow A = \frac{1}{9},\ B = 0,\ D = 0,\ C = -\frac{1}{9}$$

$$\Rightarrow \frac{1}{s^4+9s^2} = \frac{1}{9}\frac{1}{s^2} - \underbrace{\frac{1}{9}\frac{1}{s^2+9}}_{\frac{1}{27}\frac{3}{s^2+9}}$$

$$\Rightarrow y(t) = \frac{1}{9} t - \frac{1}{27} \sin(3t)$$

12) $e^{x^2/y} \stackrel{!}{=} e^{1/2}$

$$\Leftrightarrow \frac{x^2}{y} = \frac{1}{2} \Leftrightarrow y = 2x^2$$