Mathematik 2 vom 2014-07-04

Musterlösungen

1) Finde einen Vektor senkrecht zur Ebene:

$$\begin{pmatrix}1-0\\2-1\\3-2\end{pmatrix} \times \begin{pmatrix}3-0\\2-1\\4-2\end{pmatrix}$$

$$= \begin{pmatrix}1\\1\\1\end{pmatrix} \times \begin{pmatrix}3\\1\\2\end{pmatrix} = \begin{pmatrix}2-1\\3-2\\1-3\end{pmatrix}$$

$$= \begin{pmatrix}1\\1\\-2\end{pmatrix}$$

Eine Gerade $\perp$ zur Ebene:

z.B. $\begin{pmatrix}x\\y\\z\end{pmatrix} = \lambda \begin{pmatrix}1\\1\\-2\end{pmatrix}$

2)

$$y = \frac{\begin{vmatrix}1&1&0&-1\\2&2&0&0\\0&3&3&0\\1&4&0&1\end{vmatrix}}{\begin{vmatrix}1&0&0&-1\\2&1&0&0\\0&4&3&0\\1&2&0&1\end{vmatrix}} = \frac{\begin{vmatrix}1&0&0&-1\\2&0&0&0\\0&3&3&0\\1&3&0&1\end{vmatrix}}{\begin{vmatrix}0&0&0&-1\\2&1&0&0\\0&4&3&0\\2&2&0&1\end{vmatrix}} = \frac{\begin{vmatrix}0&0&0&-1\\2&0&0&0\\0&3&3&0\\2&3&0&1\end{vmatrix}}{\begin{vmatrix}0&0&0&-1\\2&1&0&0\\0&4&3&0\\0&1&0&1\end{vmatrix}}$$

$0+0-6-0-0-0$

$$= \frac{3\begin{vmatrix} 0 & 0 & -1 \\ 2 & 0 & 0 \\ 2 & 3 & 1 \end{vmatrix}}{-2\begin{vmatrix} 0 & 0 & -1 \\ 4 & 3 & 0 \\ 1 & 0 & 1 \end{vmatrix}} = \frac{\cancel{3}\cdot(-6)}{-2\cdot\cancel{3}} = 3$$

$0+0+0+3-0-0$

3) $\begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \text{Kern} \iff$

$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\iff \begin{cases} x + 2y + 3z = 0 \\ \quad\;\; 4y + 5z = 0 \\ \qquad\quad\; 6z = 0 \end{cases}$$

$$\iff z = 0 \wedge y = 0 \wedge x = 0$$

Also $\text{Kern} = \{\vec{0}\}$
und Defekt $= 0$.

4) $y' = \dfrac{\sin(x)}{y}$

$\Leftrightarrow y y' = \sin(x)$

$\Leftrightarrow$ „$y\,dy = \sin(x)\,dx$"

Also $\underbrace{\int_3^{y_1} y\,dy}_{\left[\frac{y^2}{2}\right]_3^{y_1}} = \underbrace{\int_2^{x_1} \sin(x)\,dx}_{\left[-\cos(x)\right]_2^{x_1}}$.

$\Rightarrow \dfrac{y_1^2}{2} - \dfrac{9}{2} = -\cos(x_1) + \cos(2)$

$\Rightarrow y_1 = \pm\sqrt{2(\cos(2) - \cos(x_1)) + 9}$

Muss „+" sein, wegen Anfangswert.

5) $f(x) = \frac{1}{\cos(x)}$

$f'(x) = -\frac{1}{(\cos(x))^2} \cdot (-\sin(x))$

$= \frac{\sin(x)}{(\cos(x))^2}$

$f''(x) = \frac{\cos(x)(\cos(x))^2 - \sin(x) \cdot 2\cos(x) \cdot (-\sin(x))}{(\cos(x))^4}$

$f(0) = 1$

$f'(0) = 0$

$f''(0) = 1$

Also $f(0{,}01) \approx \underbrace{1 + 0 + 1 \cdot \frac{(0{,}01)^2}{2}}_{1{,}00005}$

6) $\dfrac{3}{s^2+s-2}$

$s^2+s-2=0$

$\Longleftrightarrow s=-\frac{1}{2}\pm\sqrt{\frac{1}{4}+2}$

$=-\frac{1}{2}\pm\frac{3}{2}$

$\Longleftrightarrow s=1 \vee s=-2$

$=\dfrac{3}{(s-1)(s+2)}$

$=\dfrac{A}{s-1}+\dfrac{B}{s+2}$

mit $A=\dfrac{3}{1+2}=1$

und $B=\dfrac{3}{-2-1}=-1$

Also $y(t)=e^{t}-e^{-2t}$.

7) $\vec{x} \in$ Spiegelungsebene

$$\iff (\text{Matrix})\,\vec{x} = \vec{x}$$

$$\iff \underbrace{(\text{Matrix} - \mathbb{1})}_{\begin{pmatrix} -2/9 & -4/9 & -4/9 \\ -4/9 & -8/9 & -8/9 \\ -4/9 & -8/9 & -8/9 \end{pmatrix}}\,\vec{x} = 0$$

$$\iff \begin{cases} 2x + 4y + 4z = 0 \\ 4x + 8y + 8z = 0 \\ \quad \text{---} \;\; " \;\; \text{---} \end{cases}$$

zweimal die erste Gl.

Ebenengl. z.B.

$$\cancel{2}x + \underset{2}{\cancel{4}}y + \underset{2}{\cancel{4}}z = 0$$

$$\left(\left(\iff z = -\frac{x}{2} - y \right)\right)$$

((Könnte man auch mit Vektoren schreiben, z.B.:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \lambda \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix}$$

zwei linear unabh. Vektoren die das erfüllen))

8) homogen gemacht:

$y' + y = 0$

allgemeine Lsg.: $y(x) = Ae^{-x}$

inhomogen:

$y' + y = e^{-x}$

Ansatz: $y(x) = Be^{-x}$ ↯

2. Ansatz: $y(x) = xBe^{-x}$

$\Longrightarrow Be^{-x} \cancel{- xBe^{-x}} \cancel{+ xBe^{-x}} = e^{-x}$

$\Longrightarrow B = 1$

spez. Lsg.: $y(x) = xe^{-x}$

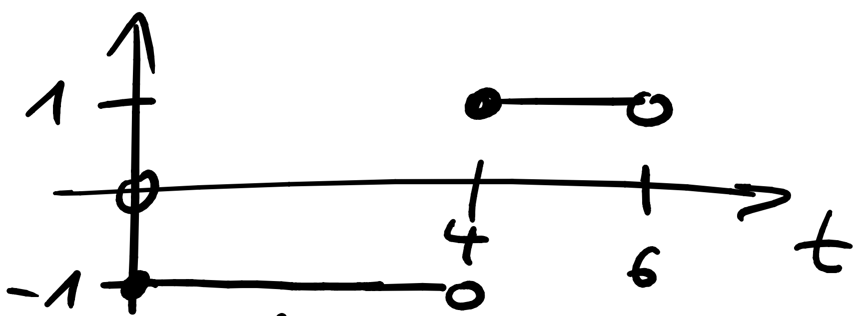
allg. Lsg.: $y(x) = Ae^{-x} + xe^{-x}$

9) Wenn man eine homogene lin. DGL mit konst. Koeff. dafür sucht, müsste sich ergeben:

$\underbrace{(\lambda - 1)(\lambda + 1)}_{\lambda^2 - 1} = 0$

Also z.B. $y'' - y = 0$.

10)



$$a_0 = \frac{\cancel{2}^{1}}{\cancel{6}_{3}}\int_0^6 f(t)\,dt = \frac{2-4}{3} = -\frac{2}{3}$$

$$b_5 = \frac{\cancel{2}^{1}}{\cancel{6}_{3}}\int_0^6 \sin\left(\cancel{2}\pi\, 5\,\frac{t}{\cancel{6}_{3}}\right) f(t)\,dt$$

$$= \frac{1}{3}\left(-\underbrace{\int_0^4 \sin\left(\pi\frac{5}{3}t\right)dt}_{\left[-\frac{3}{5\pi}\cos\left(\pi\frac{5}{3}t\right)\right]_0^4} + \underbrace{\int_4^6 \sin\left(\pi\frac{5}{3}t\right)dt}_{\left[-\frac{3}{5\pi}\cos\left(\pi\frac{5}{3}t\right)\right]_4^6}\right)$$

$$= \frac{1}{\cancel{3}}\frac{\cancel{3}}{5\pi}\left(\cos\left(\pi\frac{20}{3}\right) - 1 - \underbrace{\cos\left(\pi\cdot\frac{\cancel{30}^{10}}{\cancel{3}_{1}}\right)}_{1} + \cos\left(\pi\frac{20}{3}\right)\right)$$

$$\left(\left(= \frac{2}{5\pi}\left(\cos\left(\frac{20}{3}\pi\right) - 1\right)\right)\right)$$

11) an $(1|2|3)$:

$$f(u,v,w) = u^2 vw + u \qquad 7$$

$$\frac{\partial f}{\partial u} = 2uvw + 1 \qquad 13$$

$$\frac{\partial f}{\partial v} = u^2 w \qquad 3$$

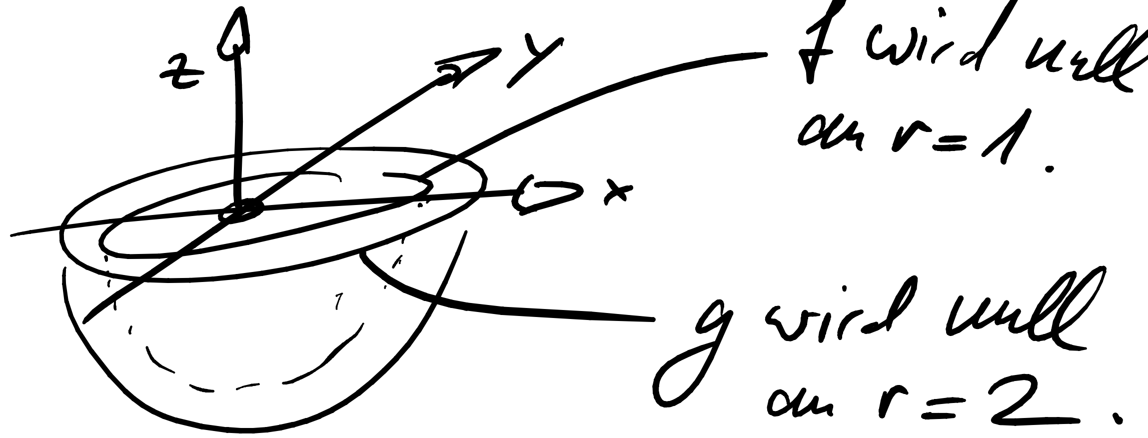
$$\frac{\partial f}{\partial w} = u^2 v \qquad 2$$

Also

$$f(1{,}01;\ 2{,}02;\ 2{,}99)$$

$$\approx \underbrace{7 + 13 \cdot 0{,}01 + 3 \cdot 0{,}02 + 2 \cdot (-0{,}01)}_{7{,}17}$$

12)



$\downarrow$

$$V = \left| \int_0^{2\pi} \left(\int_0^2 (r^2 - 4)\, r\, dr \right) d\varphi \right| - \left| \int_0^{2\pi} \left(\int_0^1 (3r^2 - 3)\, r\, dr \right) d\varphi \right|$$

$$= 2\pi \left(\left| \int_0^2 (r^3 - 4r)\, dr \right| - \left| \int_0^1 (3r^3 - 3r)\, dr \right| \right)$$

$$= 2\pi \left(\left| \left[\frac{r^4}{4} - 2r^2 \right]_0^2 \right| - \left| \left[\frac{3}{4} r^4 - \frac{3}{2} r^2 \right]_0^1 \right| \right)$$

$$= 2\pi \left(\underbrace{\left| 4 - 8 \right|}_{4} - \underbrace{\left| \frac{3}{4} - \frac{3}{2} \right|}_{\frac{3}{4}} \right)$$

$$= 2\pi \cdot 3\tfrac{1}{4}$$