Mathematik 1
2014-09-13
Musterlösungen

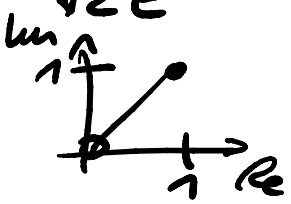
1) $\log_3(x^2-7) = 2 \iff x^2 - 7 = 9$

$\iff x^2 = 16 \iff x = \pm 4$

2) $z^2 - 2z - i = 0$

$\iff z = 1 \pm \sqrt{\underbrace{1+i}_{\sqrt{2}e^{i\pi/4}}} = 1 \pm \sqrt[4]{2}\, e^{i\pi/8}$

$= 1 \pm \sqrt[4]{2}\cos\frac{\pi}{8} \pm i\sqrt[4]{2}\sin\frac{\pi}{8}$



3) $\dfrac{n(3+4) + \sin(n^2)}{e^{-n} + n^2} = \dfrac{1(3+\frac{4}{n}) + \frac{1}{n^2}\sin(n^2)}{\frac{1}{n^2}e^{-n} + 1} \to 3$

4) $\dfrac{d\sqrt{\frac{\sin(x)-2}{x}}}{dx} = \dfrac{1}{2\sqrt{\frac{\sin(x)-2}{x}}} \cdot \dfrac{\cos(x)x - (\sin(x)-2)}{x^2}$

5) $\displaystyle\int_3^4 \frac{\cos(1/x)}{x^2}\,dx = -\int_{1/3}^{1/4} \cos(u)\,du = -\left[\sin(u)\right]_{1/3}^{1/4}$

$= -\sin\left(\frac{1}{4}\right) + \sin\left(\frac{1}{3}\right)$

$u = \frac{1}{x}$

$du = -\frac{1}{x^2}dx$

6) $L = \int_0^1 \sqrt{1+\left(\frac{3}{2}x^{\frac{1}{2}}\right)^2}\,dx$

$= \int_0^1 \sqrt{1+\frac{9}{4}x}\,dx = \left[\frac{2}{3}\left(1+\frac{9}{4}x\right)^{\frac{3}{2}}\cdot\frac{4}{9}\right]_0^1$

$= \frac{8}{27}\left(\left(1+\frac{9}{4}\right)^{3/2}-1\right)$

$\left(= \frac{8}{27}\left(\left(\frac{13}{4}\right)^{3/2}-1\right)\right)$

7) $|x^3-5| > x^3 \iff x^3 \geq 5 \wedge x^3-5 > x^3$

$\vee\ x^3 < 5 \wedge 5-x^3 > x^3$

$\iff x^3 \geq 5 \wedge -5 > 0$

$\vee\ x^3 < 5 \wedge 5 > 2x^3$

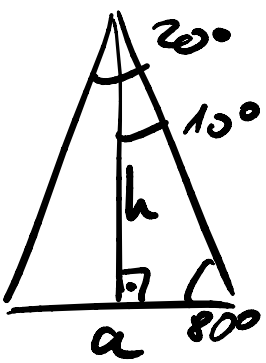
$\iff x^3 < 5 \wedge x < \sqrt[3]{\frac{5}{2}} \iff x < \sqrt[3]{\frac{5}{2}}$

d.h. $\mathbb{L} = (-\infty; \sqrt[3]{\frac{5}{2}})$

8) $C = \binom{7}{4}\cdot(-1)^3 = -\frac{7\cdot 6\cdot 5\cdot 4}{4\cdot 3\cdot 2\cdot 1}$

$= -35$

9)



$\text{Fläche} = \frac{1}{2}ah$

$= \frac{1}{2}a\frac{a}{2}\tan(80°)$

$= \frac{1}{4}\tan(80°)\cdot a^2 \overset{!}{=} 12$

$\Rightarrow a = \sqrt{48/\tan(80°)}$

10) Grad 2 muss gehen:

ax^2+bx+c

↑ muss 2 sein, wegen (0|2)

$$\Rightarrow \begin{cases} a\cdot 1^2+b\cdot 1+2=1 & \text{I} \\ a\cdot 3^2+b\cdot 3+2=1 & \text{II} \end{cases}$$

II-I: $8a+2b=0 \longrightarrow b=-\frac{4}{3}$

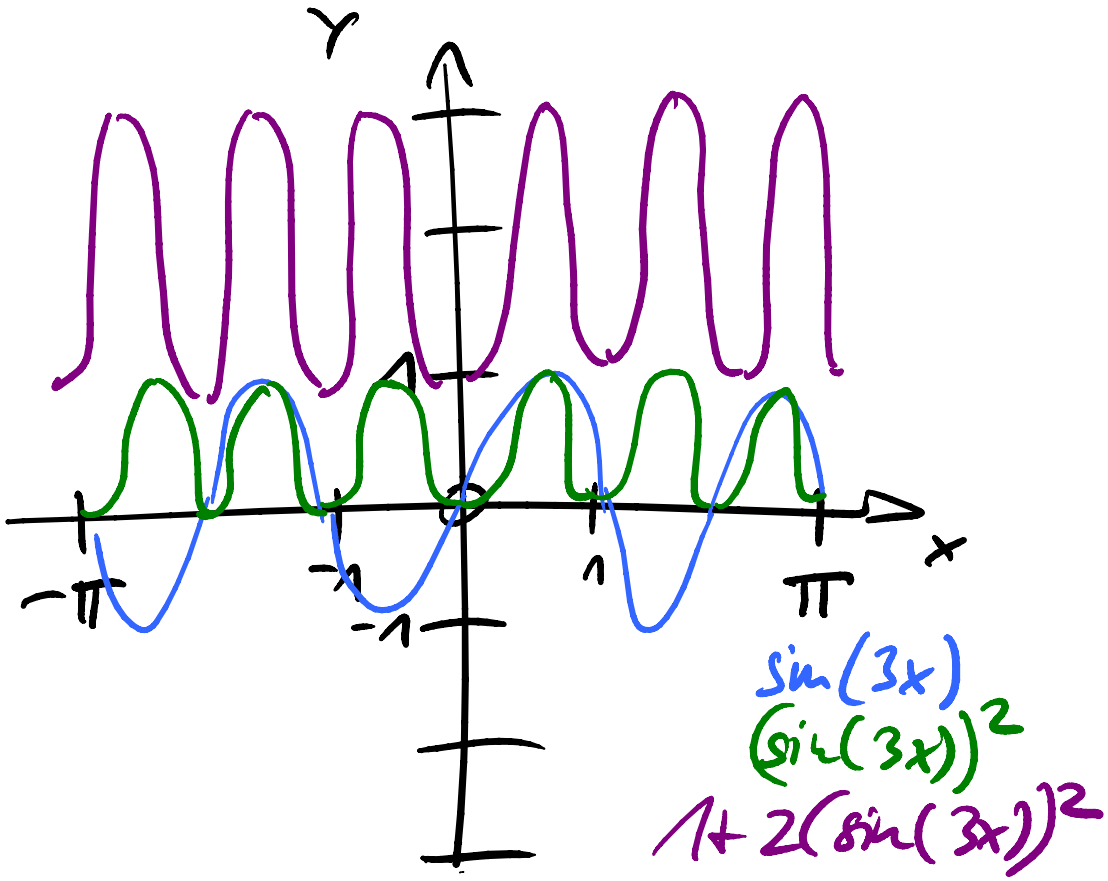
II-3I: $6a-4=-2 \Rightarrow a=\frac{1}{3}$

Also z.B. $\frac{1}{3}x^2-\frac{4}{3}x+2$

Weiteres Polynom z.B.:

$\% + x(x-1)(x-3)$

11)



12)

$$E[X^4] = \int_{-1}^{2} x^4 \, p(x) \, dx$$

$p(x) = \text{const}$

und $\int_{-1}^{2} \text{const} = 1$, also $p(x) = \frac{1}{3}$

$$= \frac{1}{3} \int_{-1}^{2} x^4 \, dx = \frac{1}{3} \left[\frac{x^5}{5} \right]_{-1}^{2}$$

$$= \frac{1}{15} \left(32 - (-1) \right) = \frac{33}{15} = 2\frac{1}{5}$$