Mathematik I

2014-01-28

① $\ln \sqrt{x^2+7} = 5$

$\Longleftrightarrow \sqrt{x^2+7} = e^5$

$\Longleftrightarrow x^2+7 = e^{10}$

$\Longleftrightarrow x^2 = e^{10}-7$

$\Longleftrightarrow x = \pm\sqrt{e^{10}-7}$

② $\dfrac{x^2}{1+x} > x^2$

$\Longleftrightarrow 1+x > 0 \wedge \dfrac{x^2}{1+x} > x^2$ größer!

$\vee\ 1+x < 0 \wedge \dfrac{x^2}{1+x} > x^2$

$\Longleftrightarrow x > -1 \wedge \cancel{x^2} > x^2(\cancel{1}+x)$ kleiner!

$\vee\ x < -1 \wedge \cancel{x^2} < x^2(\cancel{1}+x)$

$\Longleftrightarrow x > -1 \wedge 0 > x^3 \vee \underbrace{x < -1 \wedge 0 < x^3}_{\text{unmöglich}}$

$\Longleftrightarrow -1 < x < 0$ d.h. $\mathbb{L} = (-1;0)$

③

$$\frac{x-3}{\underbrace{x^2-x-6}} = \frac{\cancel{x-3}^{\,1}}{\cancel{(x-3)}(x+2)} = \frac{1}{x+2}$$

$x^2-x-6 = 0$

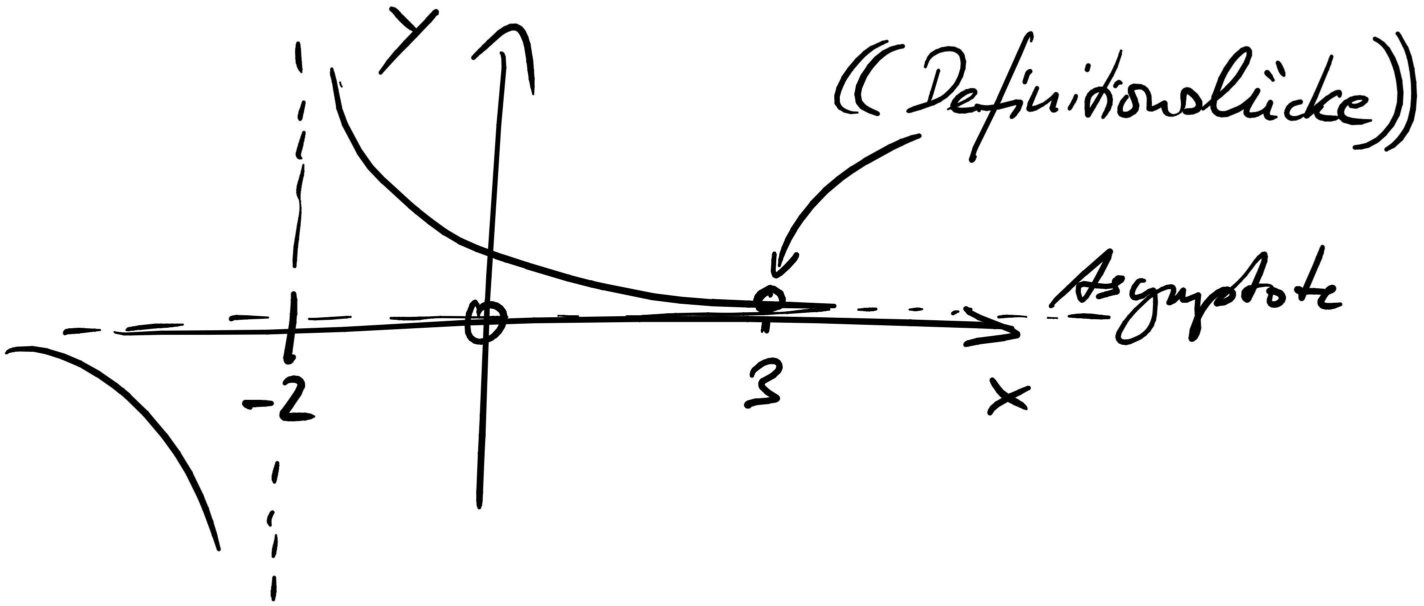
$\Longleftrightarrow x = \frac{1}{2} \pm \sqrt{\frac{1}{4}+6} = \frac{1}{2} \pm \sqrt{\frac{25}{4}}$

$\Longleftrightarrow x = 3 \lor x = -2$

Also: Polstelle bei $x = -2$,

keine Nullstelle,

Asymptote $y = 0$ für $x \to \pm\infty$



④ $$\frac{n^3+1}{e^{-n}+\sqrt{4n^6+7}} = \frac{1+\frac{1}{n^3}}{\frac{e^{-n}}{n^3}+\sqrt{\frac{4n^6+7}{n^6}}}$$

$$= \frac{\overbrace{1+\frac{1}{n^3}}^{\to 1}}{\underbrace{\frac{e^{-n}}{n^3}}_{\to 0}+\underbrace{\sqrt{\underbrace{4+\frac{7}{n^6}}_{\to 4}}}_{\to 2}} \longrightarrow \frac{1}{2}$$

⑤ $$\frac{d}{dx}\sqrt{\frac{x}{2+e^x}}$$

$$= \frac{1}{2\sqrt{\frac{x}{2+e^x}}}\,\frac{1\cdot(2+e^x)-x\cdot e^x}{(2+e^x)^2}$$

⑥ $$\int_3^5 \frac{\cos(1/x)}{x^2}\,dx$$

$$u = \frac{1}{x}$$
$$du = -\frac{1}{x^2}\,dx$$

$$= -\int_{1/3}^{1/5} \cos(u)\,du = \Big[-\sin(u)\Big]_{1/3}^{1/5} = -\sin\left(\tfrac{1}{5}\right)+\sin\left(\tfrac{1}{3}\right)$$

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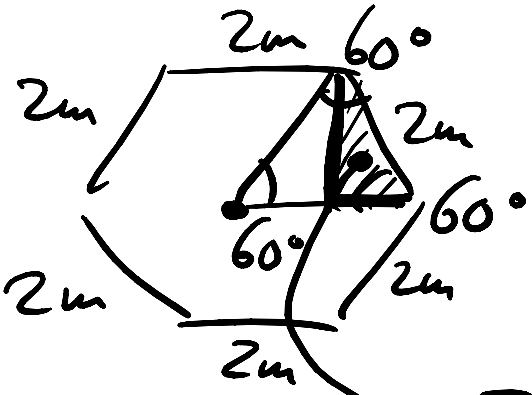

$$\text{Fläche} = \frac{1}{\cancel{2}} \, 1\,\text{m} \cdot \cancel{2}\,\text{m} \cdot \cos(30^\circ)$$

$$= \frac{\sqrt{3}}{2}\,\text{m}^2$$

$$\text{Gesamtfläche} = 12 \cdot \frac{\sqrt{3}}{2}\,\text{m}^2 = 6\sqrt{3}\,\text{m}^2$$

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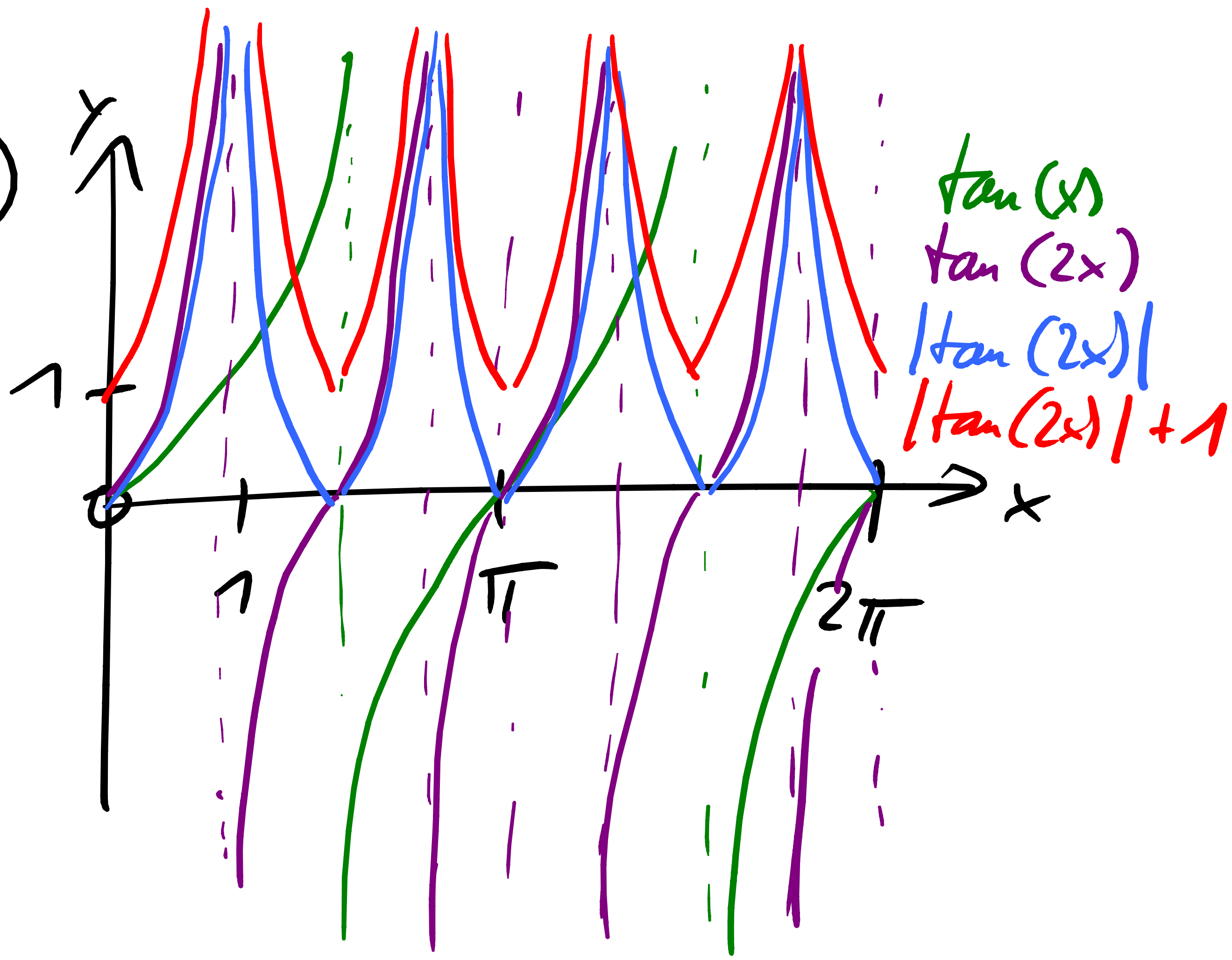

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$$z^6 - 6z^3 + 5 = 0$$

$$\Longleftrightarrow (z^3)^2 - 6z^3 + 5 = 0$$

$$\Longleftrightarrow z^3 = 3 \pm \sqrt{9-5}$$

$$\Longleftrightarrow z^3 = 5 \vee z^3 = 1$$

Also:

Länge	Winkel
1	0°
"	120°
"	240°
$\sqrt[3]{5}$	0°
"	120°
"	240°

so oder so

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$$P = \underbrace{\binom{6}{2}}_{6\cdot 5/2\cdot 1} \cdot \frac{42}{49} \cdot \frac{41}{48} \cdot \frac{40}{47} \cdot \frac{39}{46} \cdot \frac{7}{45} \cdot \frac{6}{44} = \frac{\binom{42}{4}\binom{7}{2}}{\binom{49}{6}}$$

(11) $P(\{X=1\})$ nennen wir p.

Dann: $E[X] = 1p + 2(1-p)$

$$E[X^2] = 1p + 4(1-p)$$

$$\frac{3}{10} \overset{!}{=} \sigma$$

$$\Rightarrow \frac{9}{100} = \sigma^2 = E[X^2] - (E[X])^2$$

$$= 1p + 4(1-p) - (\underbrace{1p + 2(1-p)}_{2-p})^2$$

$$= p + \cancel{4} - \cancel{4p} - (\cancel{4} - \cancel{4p} + p^2)$$

$$= p - p^2$$

$$\Rightarrow p^2 - p + \frac{9}{100} = 0$$

$$\Rightarrow p = \frac{1}{2} \pm \sqrt{\frac{1}{4} - \frac{9}{100}} = \frac{1}{2} \pm \sqrt{\frac{25-9}{100}}$$

$$= \frac{1}{2} \pm \frac{4}{10} \Rightarrow \cancel{p = \frac{1}{10}} \vee p = \frac{9}{10}$$ „1 häufiger"

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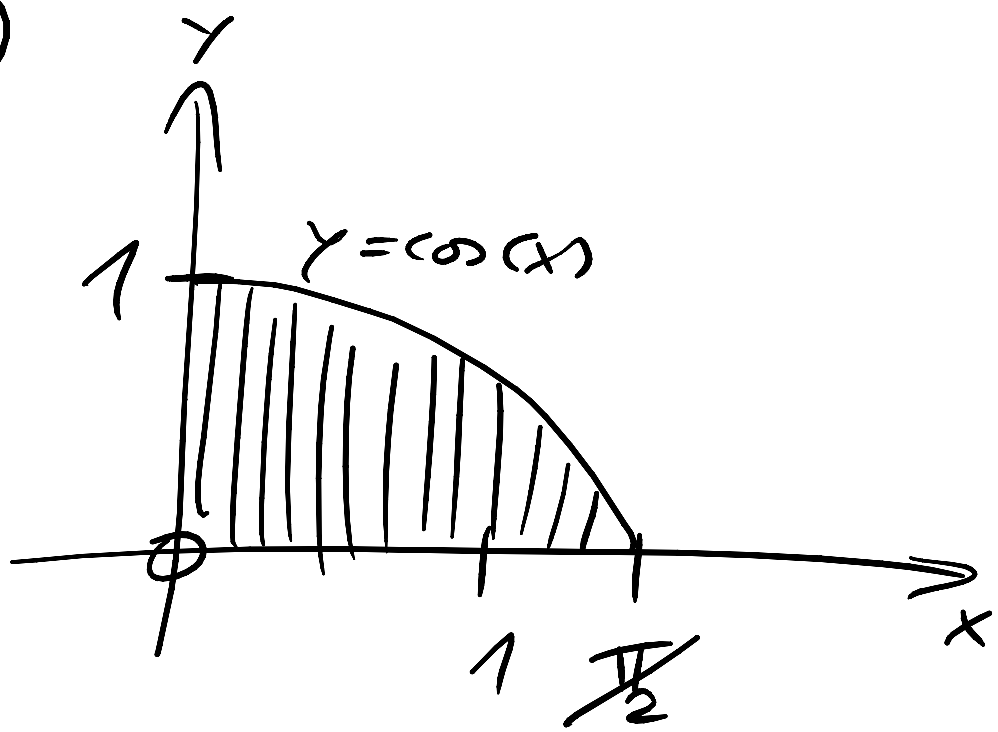

$$x_s = \frac{\int_0^{\pi/2} x \cos(x)\, dx}{\int_0^{\pi/2} \cos(x)\, dx}$$

$$\boxed{\begin{array}{cc} x & \cos(x) \\ \downarrow & \uparrow \\ 1 & \sin(x) \end{array}}$$

$$= \frac{\left[x \sin(x)\right]_0^{\pi/2} - \int_0^{\pi/2} 1 \cdot \sin(x)\, dx}{\left[\sin(x)\right]_0^{\pi/2}}$$

$$= \frac{\frac{\pi}{2} \cdot 1 - 0 - \left[-\cos(x)\right]_0^{\pi/2}}{1 - 0}$$

$$= \frac{\pi}{2} - (-0 - -1)$$

$$= \frac{\pi}{2} - 1$$