Mathematik II

2013-07-03

(1) Ebenengleichung:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 4 \\ 3 \\ 2 \end{pmatrix}$$

Schnittmenge mit xy-Ebene: $z = 0$

$$\Longleftrightarrow 0 = 3 + 2\lambda + 2\mu$$

$$\Longleftrightarrow \mu = -\lambda - \frac{3}{2}$$

Also
$$x = 1 + 2\lambda + \left(-\lambda - \frac{3}{2}\right) \cdot 4 = -2\lambda - 5$$
$$y = 2 + 2\lambda + \left(-\lambda - \frac{3}{2}\right) \cdot 3 = -\lambda - 2\tfrac{1}{2}$$
$$z = 0 \quad (\text{klar!})$$

$$\Rightarrow \text{Schnittmenge} = \text{Gerade } \begin{pmatrix} -5 \\ -2\frac{1}{2} \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ -1 \\ 0 \end{pmatrix}$$
$$\left(= \text{Gerade } \begin{pmatrix} -2 \\ -1 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} \right)$$

(2)

$$\begin{array}{ccccc} \oplus & \ominus & \oplus & \ominus & \\ \hline 0 & 1 & 5 & 0 & \\ 2 & 2 & 1 & 0 & \oplus \\ 0 & 0 & -7 & 2 & \ominus \\ 2 & 3 & 2 & 0 & \end{array} = -2 \begin{vmatrix} 0 & 1 & 5 \\ 2 & 2 & 1 \\ 2 & 3 & 2 \end{vmatrix}$$

$$= -2\,(0 + 2 + 30 - 20 - 0 - 4) = -16$$

(3) Kurz: $y' = 5(x^2+1)$

$$\Rightarrow y = 5\left(\frac{x^3}{3} + x\right) + C$$

$$y(3) \overset{!}{=} 2 \Rightarrow 2 = 5\left(\frac{3^3}{3} + 3\right) + C$$

$$\Rightarrow C = 2 - 5(9+3) = -58$$

Lang: $\frac{y'}{x^2+1} = 5$

$$\Leftrightarrow y' = 5(x^2+1)$$

$$\Leftrightarrow \text{„} dy = 5(x^2+1)\,dx \text{“}$$

Also
$$\underbrace{\int_2^{y_E} dy}_{y_E - 2} = \underbrace{\int_3^{x_E} 5(x^2+1)\,dx}_{\underbrace{5\left[\frac{x^3}{3} + x\right]_3^{x_E}}_{5\left(\frac{x_E^3}{3} + x_E - 9 - 3\right)}}$$

$$\Rightarrow y_E = 2 + 5\left(\frac{x_E^3}{3} + x_3 - 12\right)$$

(4) $f(x) = \sqrt[5]{x}$; $f'(x) = \frac{x^{-4/5}}{5}$;

$$f''(x) = -\frac{4}{25}x^{-9/5}$$

$$f(1) = 1;\ f'(1) = \frac{1}{5}$$

↓

$$\Rightarrow \sqrt[5]{1{,}01} \approx 1 + \frac{1}{5} \cdot 0{,}01 = 1{,}002$$

$$|\text{Fehler}| \leq \underbrace{\left(\max_{1 \leq x \leq 1{,}01} \left| -\frac{4}{25} \underset{\substack{\uparrow \\ \text{fallend}}}{x}^{-9/5} \right|\right)}_{\frac{4}{25}} \cdot \underbrace{\frac{(1{,}01-1)^2}{2}}_{\frac{0{,}0001}{2}}$$

$$\frac{4 \cdot 0{,}0001}{25 \cdot 2} \cdot \frac{2}{2} = \frac{8 \cdot 0{,}0001}{100}$$

$$= 0{,}000008$$

5 $$b_4 = \frac{2}{6} \int_0^6 \sin\left(\cancel{2}\pi\, 4 \frac{t}{\cancel{6}_3}\right) f(t)\, dt$$

$$= \frac{1}{3} \left(\underbrace{\int_0^3 \sin\left(\frac{4\pi}{3} t\right) \cdot 2\, dt}_{2\left[-\frac{3}{4\pi} \cos\left(\frac{4\pi}{3} t\right)\right]_0^3} + \int_3^6 \sin\left(\frac{4\pi}{3} t\right) \cdot \underbrace{(-2)\, dt}_{= \dots = 0} \right)$$

$$\underbrace{2\left[-\frac{3}{4\pi} \cos\left(\frac{4\pi}{3} t\right)\right]_0^3}_{-\frac{3}{4\pi}\left(\underbrace{\cos(4\pi) - \cos(0)}_{=0}\right)}$$

((Aber f ist <u>nicht</u> gerade!))

$= 0$ ((Hätte man auch direkt sehen können!))

⑥ $$\frac{\partial f(x,y)}{\partial x} = (2x+1)e^{\cdots}$$

$$\frac{\partial f(x,y)}{\partial y} = 2y\, e^{\cdots}$$

Also nur dann horizontale Tangentialebene, wenn $x=-\frac{1}{2}, y=0$.

$$\frac{\partial^2 f(x,y)}{\partial x^2} = 2e^{\cdots} + (2x+1)^2 e^{\cdots}$$

$$\frac{\partial^2 f(x,y)}{\partial y^2} = 2e^{\cdots} + (2y)^2 e^{\cdots}$$

$$\frac{\partial^2 f(x,y)}{\partial x \partial y} = (2x+1)\, 2y\, e^{\cdots}$$

In $(x|y) = (-\frac{1}{2}|0)$ ist

die Hesse-Matrix =

$$\begin{pmatrix} \left(2+\left(2\cdot\left(-\frac{1}{2}\right)+1\right)^2\right)e^{\cdots} & \ldots 0 \ldots \\ \ldots 0 \ldots & \left(2+(2\cdot 0)^2\right)e^{\cdots} \end{pmatrix}$$

$$= \begin{pmatrix} 2e^{\cdots} & 0 \\ 0 & 2e^{\cdots} \end{pmatrix}$$ zwei pos. Eigenwerte

$\Rightarrow$ lok. Min.

(7) $\begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 3 & -1 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$

Summe

Matrix hat den Rang 2, nicht 3.
Also typischerweise keine Lösung.

Alternativ: $\begin{vmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 3 & -1 & 2 \end{vmatrix} = 0 \Rightarrow \uparrow$

(8)

$$\begin{cases} \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = 2\begin{pmatrix} 1 \\ 1 \end{pmatrix} \\ \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 1 \\ -1 \end{pmatrix} = 3\begin{pmatrix} 1 \\ -1 \end{pmatrix} \end{cases}$$

$$\Longleftrightarrow \begin{cases} a + b = 2 \\ c + d = 2 \\ a - b = 3 \\ c - d = -3 \end{cases}$$

Summe & Differenz bilden

$$\Longleftrightarrow a = \frac{5}{2} \wedge b = -\frac{1}{2}$$
$$\wedge\, c = -\frac{1}{2} \wedge d = \frac{5}{2}$$

(9) Spez. Lsg: $y = 1$

Allg. Lsg. von $y'' + 2y' + y = 0$:

Ansatz $y = e^{\lambda x} \Rightarrow \lambda^2 + 2\lambda + 1 = 0$

$$\Rightarrow \lambda = -1 \pm \sqrt{1-1} = -1$$

Doppelte Nullstelle!

Also $y(x) = Ae^{-x} + Bxe^{-x}$

Allg. Lsg. der urspgl. DGL:

$$y(x) = Ae^{-x} + Bxe^{-x} + 1$$

(10) $y'' + ay' + 3y = 0$

Ansatz: $y = e^{\lambda x}$

$$\Rightarrow \lambda^2 + a\lambda + 3 = 0$$

$$\Rightarrow \lambda = \underbrace{-\frac{a}{2}}_{<0,\ \text{damit Abklingen}} \pm \sqrt{\underbrace{\frac{a^2}{4} - 3}_{<0,\ \text{damit sinusförmige Schwingungen}}}$$

Also $-\frac{a}{2} < 0 \wedge \frac{a^2}{4} - 3 < 0 \Leftrightarrow a > 0 \wedge a^2 < 12$

$$\Leftrightarrow 0 < a < 2\sqrt{3}$$

(11) Partialbruchzerlegung

$$\frac{s+1}{s^3} = \frac{A}{s^3} + \frac{B}{s^2} + \frac{C}{s}$$

$$= \frac{A + Bs + Cs^2}{s^3}$$

$$\Rightarrow A = 1 \wedge B = 1 \wedge C = 0$$

$$\Rightarrow \frac{s+1}{s^3} = \frac{1}{s^3} + \frac{1}{s^2}$$

$$\begin{array}{cc} \Big\downarrow \mathcal{L}^{-1} & \Big\downarrow \mathcal{L}^{-1} \\ \underline{\frac{t^2}{2}} & t \end{array}$$

(12) $$V = \int_{-\pi/2}^{\pi/2} \left(\int_0^3 \frac{\cancel{r} \cos\varphi}{\cancel{r^2}} \cancel{r}\, dr \right) d\varphi$$

$$= 3 \int_{-\pi/2}^{\pi/2} \cos\varphi \; d\varphi$$

$$= 3\left[\sin\varphi\right]_{-\pi/2}^{\pi/2}$$

$$= 6$$