Mathematik I

2013-09-19

1) $\sqrt[3]{5^{x+7}+3} = 2$

$\Leftrightarrow 5^{x+7}+3 = 8$

$\Leftrightarrow 5^{x+7} = 5$

$\Leftrightarrow x+7 = 1$

$\Leftrightarrow x = -6$

2) $2x \leq |x-7|$

$\Leftrightarrow x-7 \geq 0 \wedge 2x \leq |x-7|$

$\vee\ x-7 < 0 \wedge 2x \leq |x-7|$

$\Leftrightarrow x \geq 7 \wedge 2x \leq x-7$

$\vee\ x < 7 \wedge 2x \leq -(x-7)$

$\Leftrightarrow x \geq 7 \wedge x \leq -7$ (leere Menge)

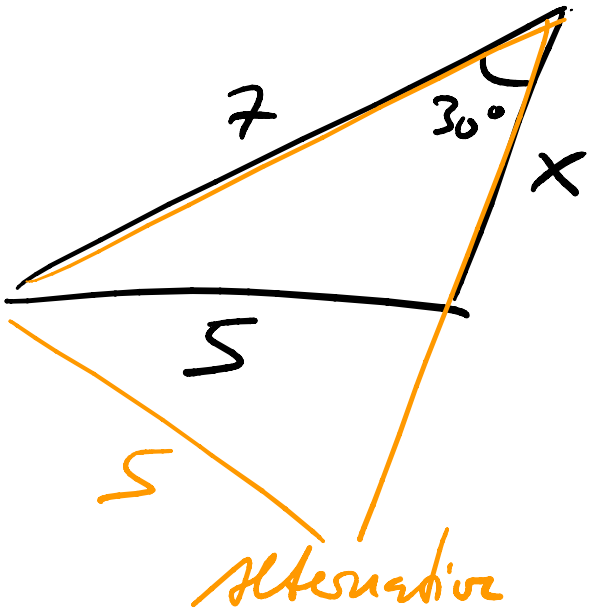
$\vee\ x < 7 \wedge 3x \leq 7$

$\Leftrightarrow x \leq \frac{7}{3}$

$x \leq \frac{7}{3}$

$\mathbb{L} = (-\infty, \frac{7}{3}]$

3)



Cosinussatz:

$$5^2 = 7^2 + x^2 - 2 \cdot 7 \cdot x \cdot \cos(30°)$$

$$\Updownarrow$$

$$x^2 - 2 \cdot 7 \cdot x \cdot \cos(30°) \underbrace{+7^2 - 5^2}_{24} = 0$$

$$x = 7\cos(30°) \pm \sqrt{(7\cos(30°))^2 - 24}$$

nicht eindeutig!

4) $\underbrace{z^4 - 2z^2 + 1}_{(z^2-1)^2} = 0$

$$\Leftrightarrow z^2 = 1 \Leftrightarrow z = 1 \vee z = -1$$

$1 + 0i$ $\quad$ $-1 + 0i$

5) $$\frac{d\left(e^x \ln(\sin(x))\right)^3}{dx}$$

$$= 3\left(e^x \ln(\sin(x))\right)^2 \cdot \left(e^x \ln(\sin(x)) + e^x \frac{\cos(x)}{\sin(x)}\right)$$

6) $P(\{X=0\}) = \frac{1}{4}$

$P(\{X=1\}) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

$P(\{X=2\}) = \frac{1}{4}$

$E[X] = \frac{1}{4} \cdot 0 + \frac{1}{2} \cdot 1 + \frac{1}{4} \cdot 2 = 1$

$E[X^2] = \frac{1}{4} \cdot 0^2 + \frac{1}{2} \cdot 1^2 + \frac{1}{4} \cdot 2^2 = \frac{3}{2}$

$$\sigma = \sqrt{\frac{3}{2} - 1} = \frac{1}{\sqrt{2}}$$

7) zum Beispiel:

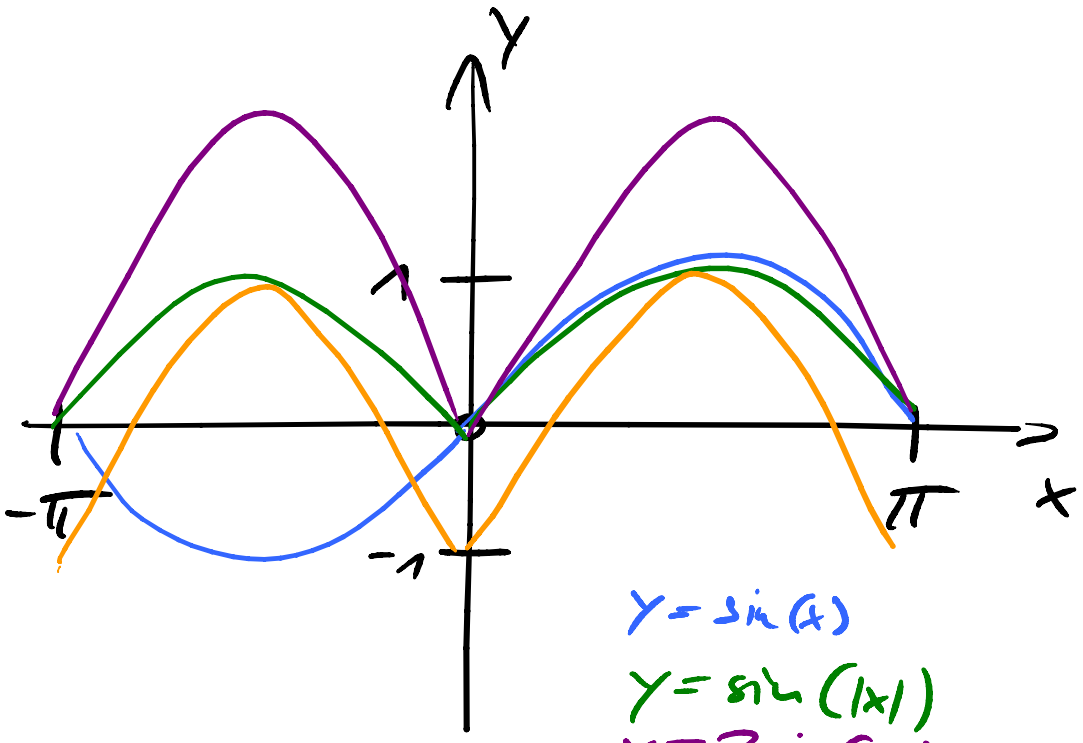
$$f(x) = x^2 + \frac{a}{(x-2)^2}$$

mit einem passenden a.

$$0 \stackrel{!}{=} f(1) = 1^2 + \frac{a}{(1-2)^2} = 1 + a.$$

Also wähle $a = -1$.

8)



$y = \sin(x)$

$y = \sin(|x|)$

$y = 2\sin(|x|)$

$y = 2\sin(|x|) - 1$

9) $\lim\limits_{x \downarrow 0} x^3 \ln(x^4) = \lim\limits_{u \to \infty} (e^{-u})^3 \ln((e^{-u})^4)$

$e^{-3u} \cdot (-4u)$

$\frac{-4u}{e^{3u}}$

wächst schneller als

$= 0$

10) $\int_0^{2\pi} x^2 \cos(x)\,dx$

($x^2 \to 2x$, $\cos(x) \to \sin(x)$)

$= \underbrace{\left[x^2 \cdot \sin(x)\right]_0^{2\pi}}_{\equiv \cdot 0 - 0^2 \cdot \equiv} - \int_0^{2\pi} \underbrace{2x}_{\to 2} \cdot \underbrace{\sin(x)\,dx}_{\to -\cos(x)}$

$= -\left(\underbrace{\left[2x \cdot (-\cos(x))\right]_0^{2\pi}}_{4\pi \cdot (-1) - 2 \cdot 0 \cdot \equiv} - \underbrace{\int_0^{2\pi} 2 \cdot (-\cos(x))\,dx}_{0} \right)$

$= 4\pi$

11) $\boxed{?} = -\binom{10}{3} \cdot \binom{7}{4} \left(= \frac{-10!}{3!\,4!\,3!} \right)$

12) $1 \overset{!}{=} \int_0^a \sqrt{\underbrace{1 + \left(\frac{3}{2} x^{1/2}\right)^2}_{1 + \frac{9}{4}x}}\,dx$

$= \left[\frac{2}{3} \cdot \frac{4}{9} \left(1 + \frac{9}{4}x\right)^{3/2}\right]_0^a = \frac{2^3}{3^3}\left(1 + \frac{9}{4}a\right)^{3/2}$

$\Rightarrow \frac{3^3}{2^3} = \left(1 + \frac{9}{4}a\right)^{3/2} \Rightarrow \frac{9}{4} = 1 + \frac{9}{4}a \Rightarrow a = \frac{5}{9}$