Mathematik I

2013-07-03

(1) $\log_4\left(2+\sqrt{x+1}\right) = 1$

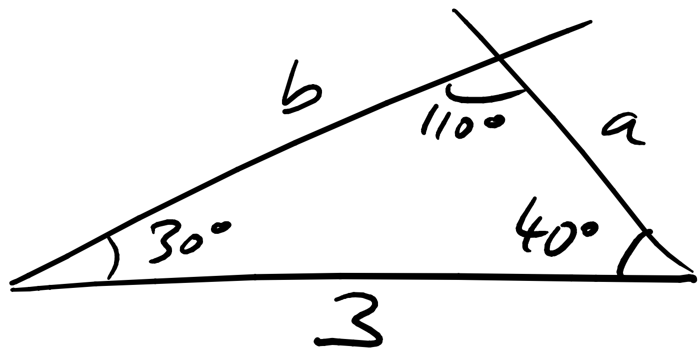
$\iff \quad 2+\sqrt{x+1} = 4^1 = 4$

$\iff \quad \sqrt{x+1} = 2$

$\iff \quad x+1 = 4$

$\iff \quad x = 3$

(2)



eindeutig!

$$a = 3 \cdot \frac{\sin 30°}{\sin 110°}, \quad b = 3 \cdot \frac{\sin 40°}{\sin 110°}$$

(3) $\dfrac{z+2i}{z-i} = i \iff z+2i = i(z-i)$

$= iz + 1$

$\iff (1-i)z = 1-2i \iff z = \dfrac{1-2i}{1-i}$

$= \dfrac{(1-2i)(1+i)}{(1-i)(1+i)} = \dfrac{1-2i+i+2}{1^2+1^2} = \dfrac{3}{2} - \dfrac{1}{2}i$

(4) $$\frac{d\,\frac{\sin(\sqrt{x})}{x^2+1}}{dx}$$

$$= \frac{\cos(\sqrt{x})\frac{1}{2\sqrt{x}}(x^2+1) - \sin(\sqrt{x})\cdot 2x}{(x^2+1)^2}$$

(5) $$\int_1^5 \frac{1}{x}\left(\ln(x)\right)^3 dx$$

$u = \ln(x)$, $\frac{du}{dx} = \frac{1}{x}$, $\frac{1}{x}dx = du$

$$= \int_{\ln(1)}^{\ln(5)} u^3\,du = \left[\frac{u^4}{4}\right]_{\ln(1)}^{\ln(5)}$$

$$= \frac{1}{4}\left(\left(\ln(5)\right)^4 - \underbrace{\left(\ln(1)\right)^4}_{0}\right)$$

(6) Poisson-Verteilung! $\lambda = 4$

$$P(\{X=0\}) = \frac{\lambda^0 e^{-\lambda}}{0!} = e^{-4}$$

7 Zähler $= (x+1)^2$

Nenner hat Nullstelle bei $x=1$.

$$(x^3 - x^2 - x + 1) : (x-1) = x^2 - 1$$
$$-(x^3 - x^2)$$
$$\overline{0 \quad -x+1}$$
$$-(-x+1)$$
$$\overline{0}$$

$$= (x+1)(x-1)$$

Also Nenner $= (x+1)(x-1)^2$

Also Funktion $= \dfrac{(x+1)^{\cancel{2}}}{\cancel{(x+1)}(x-1)^2}$

mit Partialbrüchen:

$$\ldots = \frac{A}{(x-1)^2} + \frac{B}{x-1}$$

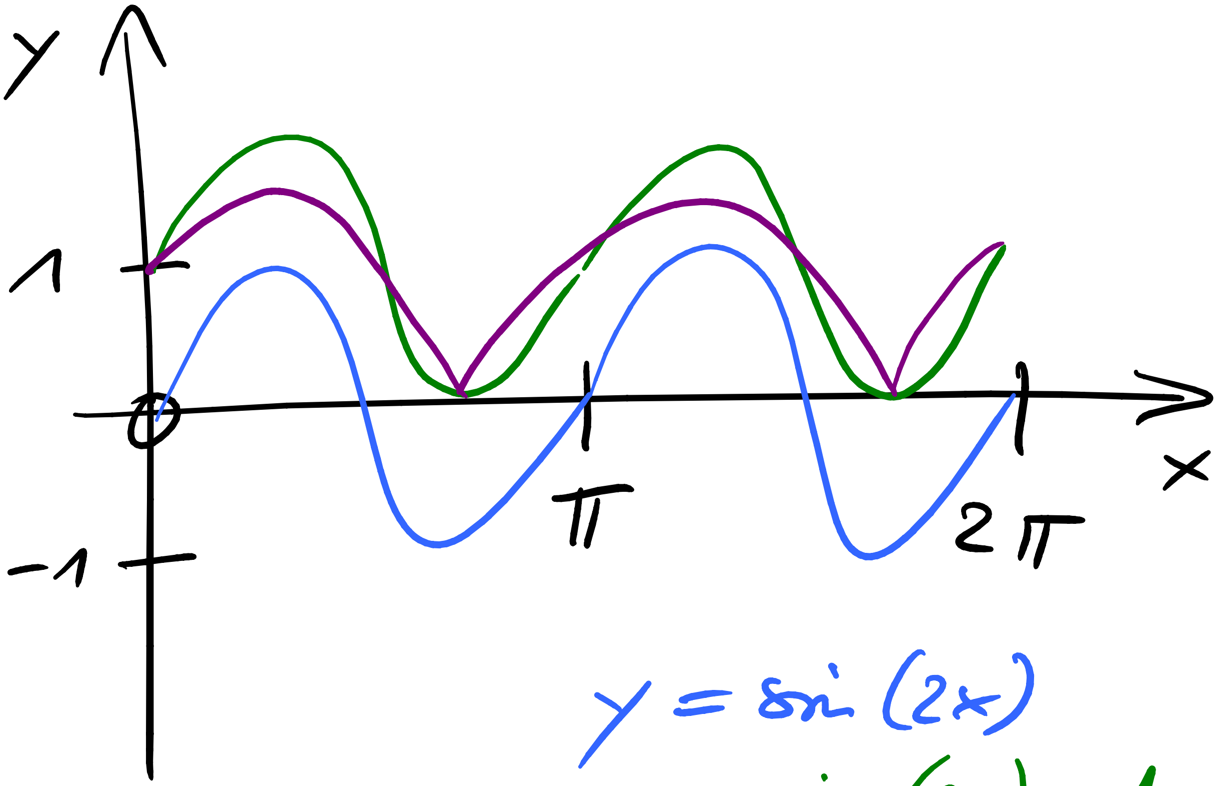
((Und nun Hauptnenner usw. oder:))

$A = 2$, wegen $\dfrac{x+1}{(x-1)^2}$ $\longrightarrow$ an $x=1$: 2

$B = 1$, damit es z.B. für $x=0$ passt:

$$\frac{0+1}{(0-1)^2} \stackrel{!}{=} \frac{2}{(0-1)^2} + \frac{B}{0-1}$$

(8)



$y = \sin(2x)$

$y = \sin(2x) + 1$

$y = \sqrt{\sin(2x) + 1}$

(9) Kurz: $x^2 \le (x+1)^2 = x^2 + 2x + 1$

$\iff 0 \le 2x + 1 \iff x \ge -\frac{1}{2}$

Lang: $x^2 \le (x+1)^2$

$\iff x \ge 0 \wedge x^2 \le (x+1)^2$

$\quad \vee\ x < 0 \wedge x^2 \le (x+1)^2$

$\iff x \ge 0 \wedge \underbrace{x \le |x+1| \overset{\text{weil } x \ge 0}{=} x+1}_{\text{immer wahr}}$

$\quad \vee\ x < 0 \wedge -x \le |x+1|$

$\iff x \ge 0$

$\quad \vee\ x < 0 \wedge (x+1 \ge 0 \wedge -x \le |x+1|$

$\qquad \vee\ x+1 < 0 \wedge -x \le |x+1|)$

$\downarrow$

$$\Leftrightarrow x \geq 0$$
$$\vee\ x < 0 \wedge \big(x \geq -1 \wedge -x \leq x+1$$
$$\vee\ x < -1 \wedge -x \leq -(x+1) \big)$$

$-1 \leq 2x$

$-x \leq -x - 1$
immer falsch

$$\Leftrightarrow x \geq 0$$
$$\vee\ x < 0 \wedge \cancel{x \geq -1} \wedge x \geq -\frac{1}{2}$$

$$\Leftrightarrow x \geq -\frac{1}{2} \text{, d.h. } \mathbb{L} = [-\tfrac{1}{2}; \infty)$$

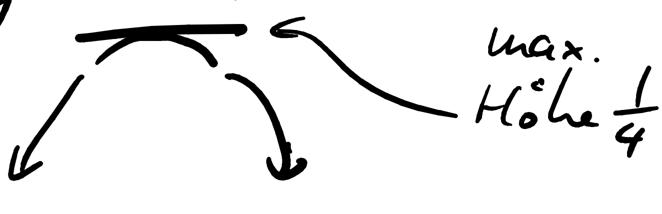
10

$$\frac{d\, e^x(1-e^x)}{dx} = e^x(1-e^x) + e^x \cdot (-e^x) = e^x(1-2e^x)$$

wird genau dann null, wenn $1 = 2e^x$

$$\Rightarrow e^x = \frac{1}{2} \Rightarrow e^x(1-e^x) = \frac{1}{4}$$

Für $x \to \pm\infty$ geht die Funktion gegen $-\infty$:



(11) Erwartungswert

$= 3p + 4(1-p)$

$= 4 - p$

Varianz

$= 3^2 p + 4^2(1-p) - (4-p)^2$

$= 9p \cancel{+16} - 16p \cancel{-16} + 8p - p^2$

$= p - p^2$

$\stackrel{!}{=} \left(\frac{1}{3}\right)^2$

$\Longleftrightarrow p^2 - p + \left(\frac{1}{3}\right)^2 = 0$

$\Longleftrightarrow p = \frac{1}{2} \pm \sqrt{\underbrace{\frac{1}{4} - \frac{1}{9}}_{\frac{9-4}{36}}} = \frac{1}{2} \pm \frac{\sqrt{5}}{6}$

(12) Volumen $= \int_0^\infty \pi \left(e^{-x}\right)^2 dx$

$= \pi \int_0^\infty e^{-2x} dx = \pi \left[\frac{e^{-2x}}{-2}\right]_0^{\text{"}\infty\text{"}}$

$= \pi \left(\frac{0}{-2} - \frac{1}{-2}\right) = \pi/2$