Mathematik 2
Klausur vom 4.7.12
Musterlösungen

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1. Vektor ⊥ Ebene z.B.

$$\begin{pmatrix}4-1\\3-1\\2-1\end{pmatrix} \times \begin{pmatrix}1-1\\2-1\\3-1\end{pmatrix} = \begin{pmatrix}3\\2\\1\end{pmatrix} \times \begin{pmatrix}0\\1\\2\end{pmatrix}$$

$$= \begin{pmatrix}3\\-6\\3\end{pmatrix}$$

Also Geradengl. z.B. $\lambda \begin{pmatrix}3\\-6\\3\end{pmatrix}$.

2. λ ist E.W. $\Longleftrightarrow$

$$0 = \begin{vmatrix}1-\lambda & 2 & 3\\ 0 & 4-\lambda & 0\\ 0 & 5 & 6-\lambda\end{vmatrix} = (1-\lambda)(4-\lambda)(6-\lambda)$$
$$+0+0-0-0-0$$

$$\Longleftrightarrow \lambda = 1 \vee \lambda = 4 \vee \lambda = 6$$

3. Ansatz: $y(x) = Ax^3 + Bx^2 + Cx + D$

$$y'(x) = 3Ax^2 + 2Bx + C$$

$$y''(x) = 6Ax + 2B$$

Einsetzen:

$$6Ax + 2B + 9Ax^2 + 6Bx + 3C \overset{!}{=} x^2$$

$$\iff \left\{\begin{array}{l} 2B + 3C = 0 \\ 6A + 6B = 0 \\ 9A = 1 \end{array}\right\} \iff \left\{\begin{array}{l} A = 1/9 \\ B = -1/9 \\ C = 2/27 \end{array}\right.$$

D egal!

Also z.B. $y(x) = \frac{x^3}{9} - \frac{x^2}{9} + \frac{2}{27}x$

4. $\frac{dy}{dx} = \frac{x}{y+1} \iff$ „$(y+1)dy = x\,dx$"

$$\Rightarrow \underbrace{\int_3^{y_1} (y+1)\,dy}_{\left[\frac{1}{2}(y+1)^2\right]_3^{y_1}} = \underbrace{\int_5^{x_1} x\,dx}_{\left[\frac{x^2}{2}\right]_5^{x_1}}$$

↓

$$\Rightarrow \frac{1}{2}(y_1+1)^2 - \frac{1}{2}4^2 = \frac{x_1^2}{2} - \frac{5^2}{2}$$

$$\Rightarrow \frac{1}{2}(y_1+1)^2 = \frac{x_1^2}{2} \underbrace{- \frac{25}{2} + 8}_{-5/2}$$

$$\Rightarrow y_1 = -1 \pm \sqrt{x_1^2 - 5}$$

muss + sein, wegen Anfangswert

5. $$\frac{s^2+1}{s^3-s^2} = \frac{A}{s^2} + \frac{B}{s} + \frac{C}{s-1}$$

Nenner $= s^2(s-1)$

$$= \frac{A(s-1) + Bs(s-1) + Cs^2}{s^2(s-1)}$$

$$\Rightarrow \left.\begin{array}{l} s^0 : 1 = -A \\ s^1 : 0 = A - B \\ s^2 : 1 = B + C \end{array}\right\} \Rightarrow \left\{\begin{array}{l} A = -1 \\ B = -1 \\ C = 2 \end{array}\right.$$

$$\Rightarrow y(t) = -t - 1 + 2e^t$$

6. Notwendig: Gradient $= \vec{0}$

$$\frac{\partial f}{\partial x} = 6x - 14y + 10$$

$$\frac{\partial f}{\partial y} = -14x + 16y + 10$$

$$\frac{\partial f}{\partial x}(3;2) = 18 - 28 + 10 = 0 \checkmark$$

$$\frac{\partial f}{\partial y}(3;2) = -42 + 16 \cdot 2 + 10 = 0 \checkmark$$

Was sagt Hesse?

$$\frac{\partial^2 f}{\partial x^2} = 6 \; ; \; \frac{\partial^2 f}{\partial y^2} = 16 \; ; \; \frac{\partial^2 f}{\partial x \partial y} = -14$$

Hesse-Matrix: $\begin{pmatrix} 6 & -14 \\ -14 & 16 \end{pmatrix}$

$$\text{Determinante} = 4 \cdot \underbrace{\begin{vmatrix} 3 & -7 \\ -7 & 8 \end{vmatrix}}_{24-49} < 0$$

$\Longrightarrow$ weder lok. Max.
noch lok. Min.

7. $$\begin{pmatrix}-1 & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}=\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$$

8. $$\left(\left(\text{Kern} = \left\{\begin{pmatrix}1\\1\\2\end{pmatrix}, \begin{pmatrix}3\\3\\5\end{pmatrix}, \begin{pmatrix}-7\\-7\\13\end{pmatrix}, \dots\right\}\right)\right)$$

Matrix z.B. $\begin{pmatrix}1 & -1 & 0\\ 1 & -1 & 0\end{pmatrix}$

9. $$f(x) = \sqrt[3]{x} \xrightarrow{x=8} 2$$

$$f'(x) = \frac{1}{3}x^{-2/3} \longrightarrow \frac{1}{3}2^{-2} = \frac{1}{12}$$

$$f''(x) = -\frac{2}{9}x^{-5/3}$$

Also $\sqrt[3]{9} \approx 2 + \frac{1}{12}\cdot 1$ mit

$$|\text{Fehler}| \leq \max_{8\leq x\leq 9} \frac{2}{9}\underbrace{x^{-5/3}}_{\leq\, 8^{-5/3}} \cdot \underbrace{\frac{(x-8)^2}{2}}_{\leq\, \frac{1}{2}}$$

fallend!

$$\leq \frac{\cancel{2}}{9}\cdot 2^{-5}\cdot\frac{1}{\cancel{2}} = \frac{1}{9\cdot 32}$$

10. $y'' - 9y = 0$

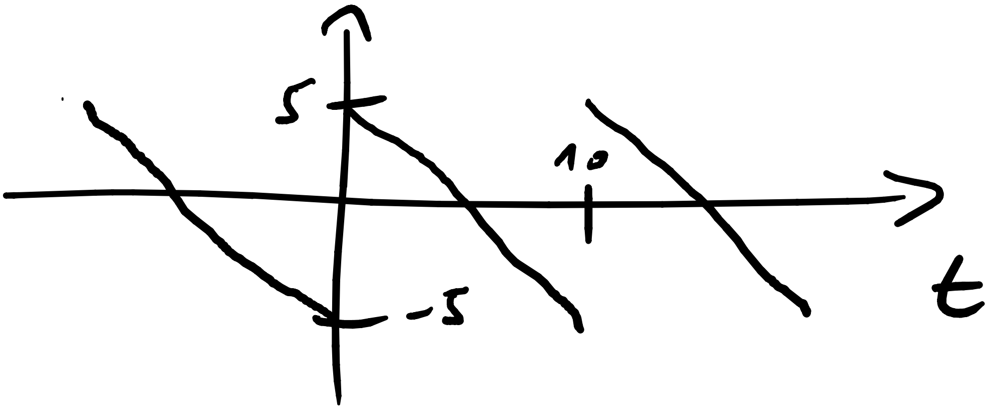
Ansatz: $y = e^{\lambda x}$

$\Rightarrow \lambda^2 - 9 = 0 \Rightarrow \lambda = \pm 3$

Also allg. Lösung:

$$y(x) = \underset{\uparrow \text{ geht \underline{nicht} } \to 0}{Ae^{3x}} + Be^{-3x}$$

11.



ungerade Funktion $\Rightarrow a_{\dots} = 0$

$$b_2 = \frac{\cancel{2}}{\underset{5}{\cancel{10}}} \int_0^{10} \sin\left(2\cdot\cancel{2}\pi \frac{t}{\underset{5}{\cancel{10}}}\right)(5-t)\,dt$$

↑ Fällt aus, wg. Periode vom Sinus

$$= -\frac{1}{5}\int_0^{10} \sin\left(2\frac{\pi t}{5}\right) t \, dt$$

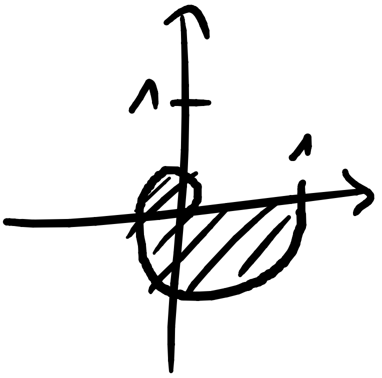
$$\begin{array}{cc} \uparrow & \downarrow \\ -\frac{5}{2\pi}\cos\left(2\frac{\pi t}{5}\right) & 1 \end{array}$$

$$= -\frac{1}{5}\left(\left[-\frac{5}{2\pi}\cos\left(2\frac{\pi t}{5}\right) t\right]_0^{10} - \underbrace{-\frac{5}{2\pi}\int_0^{10}\cos\left(2\frac{\pi t}{5}\right) dt}_{0 \text{ wg. Periode}}\right)$$

$$= \frac{1}{2\pi}\left(\cos(4\pi)\cdot 10 - 0\right)$$

$$= \frac{5}{\pi}.$$

12.



Fläche =

$$\int_0^{2\pi}\left(\int_0^{\varphi/2\pi} 1 \underset{\uparrow\,!}{r}\, dr\right) d\varphi$$

$$= \int_0^{2\pi}\left[\frac{r^2}{2}\right]_0^{\varphi/2\pi} d\varphi$$

$\downarrow$

$$= \int_0^{2\pi} \left(\frac{1}{2} \frac{\varphi^2}{(2\pi)^2} - 0 \right) d\varphi$$

$$= \frac{1}{8\pi^2} \left[\frac{\varphi^3}{3} \right]_0^{2\pi} = \frac{8\pi^3}{3 \cdot 8\pi^2} = \frac{\pi}{3}$$