Mathematik I

2012-09-19

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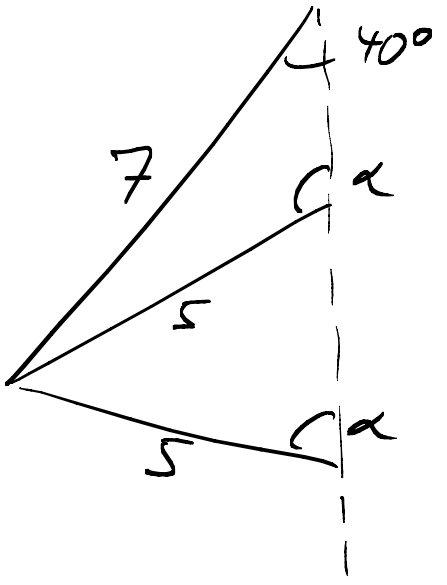
1) $\log_2(4^{x-1}) = 3$

$\Leftrightarrow 4^{x-1} = 2^3 = 4^{3/2}$

$\Leftrightarrow x-1 = 3/2 \Leftrightarrow x = 2\frac{1}{2}$

2)



Der Winkel α ist nicht eindeutig festgelegt.

$\frac{\sin\alpha}{7} = \frac{\sin 40°}{5}$

$\Rightarrow \alpha = \arcsin\left(\frac{7}{5}\cdot \sin 40°\right)$

oder $180° - \alpha$

((64° oder 116°))

3) $\underbrace{2z+z^3}_{z(2+z^2)} = 0 \Leftrightarrow z = 0 \vee \underbrace{z^2 = -2}_{\Updownarrow}$

$z = 0+0i$

$z = \pm\sqrt{2}\,i = 0 \pm \sqrt{2}\,i$

4) $$\frac{d\,\cdot}{dx} = \frac{1}{\frac{2+\sin(t)}{3+x^2}} \quad \frac{\cos(t)(3+x^2) - (2+\sin(t))\cdot 2x}{(3+x^2)^2}$$

$$\left(= \frac{3+x^2}{2+\sin(t)} \quad \frac{\cdots}{(3+x^2)^2} \right)$$

5) $x + 1 \leq x^2$

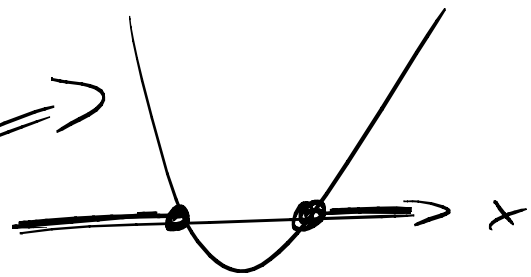
$\Leftrightarrow x^2 - x - 1 \geq 0$

↳ Wo wird das 0?

$$x = \frac{1}{2} \pm \sqrt{\frac{1}{4}+1}$$

$$= \frac{1}{2} \pm \frac{\sqrt{5}}{2}$$

nach oben geöffnete Parabel



$\Rightarrow \mathbb{L} = (-\infty; \frac{1}{2} - \frac{\sqrt{5}}{2}] \cup [\frac{1}{2} + \frac{\sqrt{5}}{2}; \infty)$

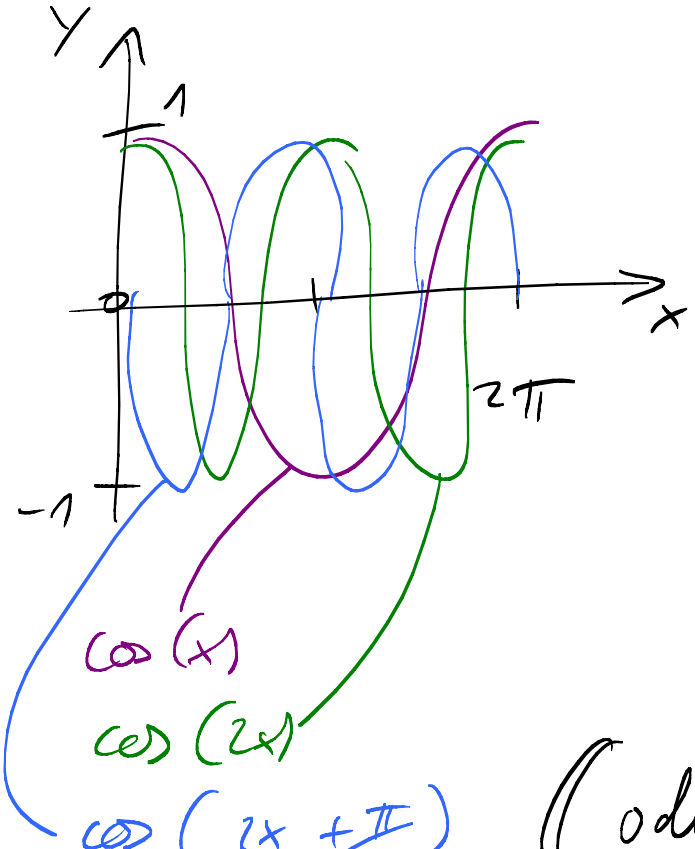
6) $x^3 - x = x(x^2 - 1)$

$$\Rightarrow \frac{x^2-1}{x^3-x} = \frac{x^2-1}{x(x^2-1)} = \frac{1}{x}$$

$\Rightarrow$ Polstelle bei $x = 0$,

keine Nullstelle

7)



((oder: ... $= \cos\left(2\left(x+\frac{\pi}{4}\right)\right)$))

8)

$$\frac{d\,e^{2x}-e^{x}}{dx} = 2e^{2x}-e^{x}$$

Lokales Minimum?

$$0 = \text{!} = e^{x}\left(2e^{x}-1\right)$$

$$\Leftrightarrow e^{x} = \frac{1}{2} \Leftrightarrow x = \ln\left(\tfrac{1}{2}\right)$$

Wert dort: $e^{2\ln(\frac{1}{2})} - e^{\ln(\frac{1}{2})} = \left(\frac{1}{2}\right)^2 - \frac{1}{2} = -\frac{1}{4}$

Für $x \to -\infty$ wird $e^{2x}-e^{x}$ zu null,

für $x \to +\infty$ wird $e^{2x}-e^{x}$ +unendlich.

Also kleinster Wert: $-\frac{1}{4}$

9) $$\int_3^5 \frac{(\ln(x))^2}{x}\,dx$$

$u = \ln(x)$

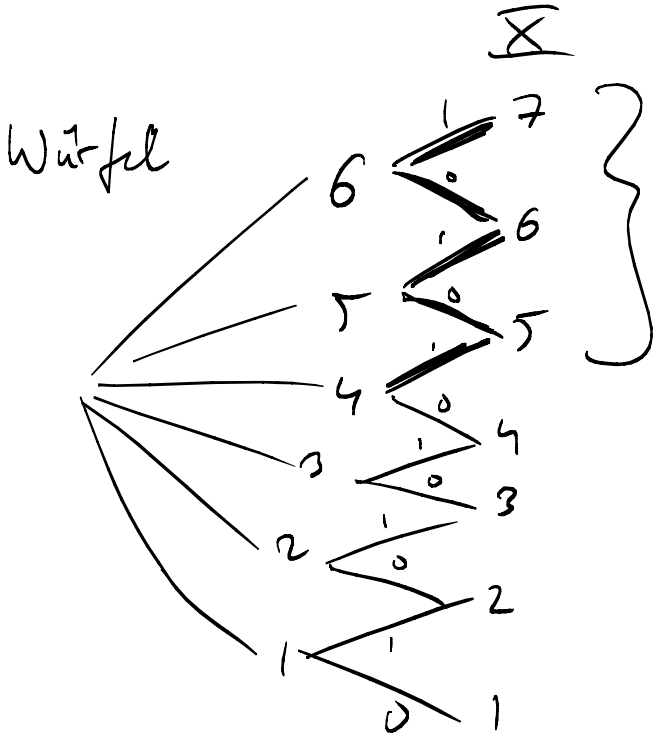
$\frac{du}{dx} = \frac{1}{x}$

$du = \frac{dx}{x}$

$$= \int_{\ln(3)}^{\ln(5)} u^2\,du$$

$$= \left[\frac{u^3}{3}\right]_{\ln(3)}^{\ln(5)} = \frac{\ln(5)^3}{3} - \frac{\ln(3)^3}{3}$$

10)



$P = \frac{5}{12}$

11)

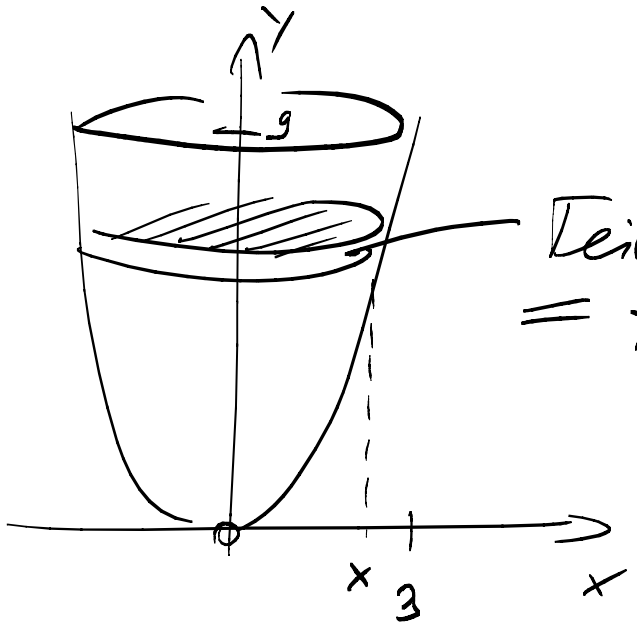
$$\sin\left(\frac{n^2-1}{n+e^n}\right) = \sin\left(\frac{1-\frac{1}{n^2}}{\frac{1}{n}+\frac{e^n}{n^2}}\right)$$

$\to 0$ (oben), 0, ∞

also Bruch $\to 0$

und damit $\sin \to 0$.

12)



Teilvolumen
$= \pi x^2 \, dy$

$$V = \int_0^g \pi \underbrace{x^2}_{=y} dy = \pi \int_0^g y \, dy = \pi \frac{g^2}{2} = \frac{g^2}{2}\pi$$

((Oder die Achsen vertauschen und mit der Formel aus der Vorlesung arbeiten.))