

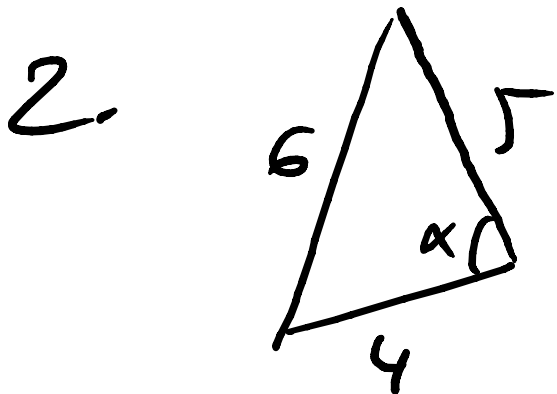
Mathematik 1

2012-07-04
Mustlösungen

1. ... $\Leftrightarrow 5^{x+2} + 1 = 8$

$\Leftrightarrow x+2 = \log_5(7)$

$\Leftrightarrow x = \log_5(7) - 2$



$$6^2 = 5^2 + 4^2 - 2 \cdot 4 \cdot 5 \cos \alpha$$

$$\Rightarrow \alpha = \arccos \frac{25 + 16 - 36}{40}$$

eindeutig!

$$= \arccos \frac{5}{8}$$

$$\text{Fläche} = \frac{1}{2} \cdot 4 \cdot 5 \cdot \sin \alpha$$

$$= 10 \cdot \sin \arccos \frac{1}{2}$$

$$\left(= 10 \cdot \sqrt{1 - \left(\frac{1}{2}\right)^2} \right)$$

$$3. \quad \underbrace{z^4 - iz^2 = 0}_{z^2(z^2 - i)}$$

$$\Leftrightarrow z = 0$$

$$\vee z = \frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}$$

$$\vee z = -\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}$$

(Wurde aus i , z.B. über Länge und Winkel)

4. Ableitung =

$$\exp(\cdot) \cdot \frac{\sqrt{x} \cos(x) - \frac{1}{2\sqrt{x}} \sin(x)}{x}$$

$$5. \quad \int_3^5 x^2 \ln(x) dx$$

\uparrow \downarrow
 $\frac{x^3}{3}$ $\frac{1}{x}$

$$= \left[\frac{x^3}{3} \ln(x) \right]_3^5 - \int_3^5 \frac{x^3}{3} \cdot \frac{1}{x} dx$$

$$= \frac{125}{3} \ln(5) - 9 \ln(3) - \left[\frac{x^3}{9} \right]_3^5$$

$$= \frac{125}{3} \ln(5) - 9 \ln(3) - 125/9 + 3$$

6. Binomialverteilung:

$$P = \underbrace{\binom{100.000}{0}}_1 \cdot \underbrace{(10^{-6})^0}_1 \cdot \underbrace{(1 - 10^{-6})^{100.000}}_{\left(\hat{=} e^{-1/10}\right)}$$

oder Poisson-Verteilung $\approx \frac{9}{10}$

$$P = \frac{\lambda^0 e^{-\lambda}}{0!} \quad \text{mit } \lambda = 10^{-6} \cdot 100.000 = 0,1$$
$$= e^{-0,1} \quad \left(\hat{=} \frac{9}{10}\right)$$

7. Ansatz: $N(t) = 1000 e^{At}$

$$\Rightarrow 10 \text{ f f f} = 1 \text{ f f f} e^{A \cdot 24 \text{ h}}$$

$$\Rightarrow A \cdot 24 \text{ h} = \ln(10)$$

$$\Rightarrow A = \frac{1}{24 \text{ h}} \cdot \ln(10)$$

$$\Rightarrow N(t) = 1000 \cdot e^{\ln(10) \cdot \frac{t}{24 \text{ h}}}$$

$$= 1000 \cdot \underset{\uparrow}{10}^{t/24 \text{ h}}$$



↳ Klätte man auch gleich drauf kommen können!

$$\Rightarrow N(1h) = 1000 \cdot 10^{1/24}$$

$$\left(\approx 1101 \right)$$

$$8. \frac{x+1}{x^2+x^3} = \frac{\overset{1}{x+1}}{x^2(\overset{1}{x+1})} = \frac{1}{x^2}$$

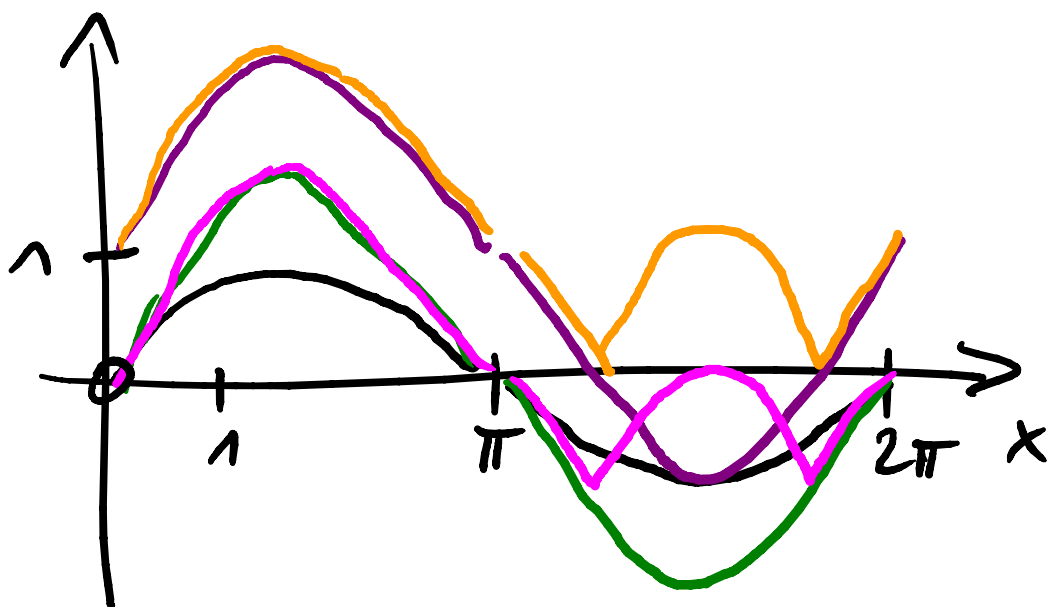
Fertig!

(Oder aufwendig =

$$\therefore = \frac{A}{x^2} + \frac{B}{x} + \frac{C}{x+1}$$

$$\dots \Rightarrow A=1, B=0, C=0$$

9.



$\sin(x)$

$2\sin(x)$

$2\sin(x)+1$

$|2\sin(x)+1|$

$|2\sin(x)+1|-1$

10. $|x| \geq (x+1)^2$

$\Leftrightarrow x \geq 0 \wedge x \geq (x+1)^2$

$\vee x < 0 \wedge -x \geq \underbrace{(x+1)^2}_{x^2+2x+1}$

$\Leftrightarrow x \geq 0 \wedge 0 \geq x^2+x+1$

$\vee x < 0 \wedge 0 \geq x^2+3x+1$



Nebenrechnung:

$$x^2 + x + 1 = 0$$

$$\Leftrightarrow x = -\frac{1}{2} \pm \sqrt{\frac{1}{4} - 1}$$

$\Rightarrow x^2 + x + 1$ ist
immer > 0

$$x^2 + 3x + 1 = 0$$

$$\Leftrightarrow x = -\frac{3}{2} \pm \underbrace{\sqrt{\frac{9}{4} - 1}}_{\sqrt{5}/2}$$

$$\Rightarrow x^2 + 3x + 1 \leq 0$$

für $\underbrace{-\frac{3}{2} - \frac{\sqrt{5}}{2} \leq x \leq -\frac{3}{2} + \frac{\sqrt{5}}{2}}$

$$\Leftrightarrow x < 0 \quad \checkmark$$

$$\Leftrightarrow x \in \left[-\frac{3}{2} - \frac{\sqrt{5}}{2}; \underbrace{-\frac{3}{2} + \frac{\sqrt{5}}{2}} \right]$$

Da dies
ist offensichtlich
 < 0

$$11. \quad P(\underline{X}=0) = \frac{1}{8} \quad zzz$$

$$P(\underline{X}=1) = \frac{3}{8} \quad zzk \text{ usw.}$$

$$P(\underline{X}=2) = \frac{3}{8} \quad zkk \text{ usw.}$$

$$P(\underline{X}=3) = \frac{1}{8} \quad kkk$$

$$E[\underline{X}] = \frac{3}{2} \quad (\text{offensichtlich oder } 0 \cdot \frac{1}{8} + 1 \cdot \frac{3}{8} + 2 \cdot \frac{3}{8} + 3 \cdot \frac{1}{8})$$

$$E[\underline{X}^2] = 0 \cdot \frac{1}{8} + 1^2 \cdot \frac{3}{8} + 2^2 \cdot \frac{3}{8} + 3^2 \cdot \frac{1}{8}$$

$$= \frac{3+12+9}{8} = \frac{24}{8} = 3$$

$$\sigma = \sqrt{3 - \left(\frac{3}{2}\right)^2}$$

$$= \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$$

$$12. \frac{\sqrt{e^{-u} + 2u^4 + u^3}}{u + 5u^2 + 7 \operatorname{sh}(u)}$$

$$= \frac{\sqrt{\frac{e^{-u}}{u^4} + 2 + \frac{1}{u}}}{\frac{1}{u} + 5 + \frac{7}{u^2} \operatorname{sh}(u)} \rightarrow \frac{\sqrt{2}}{5}$$

The diagram shows the limit process as $u \rightarrow 0$. Red circles and arrows indicate the following steps:

- The term e^{-u} in the numerator is circled, with an arrow pointing to a red 0 above it.
- The term $1/u$ in the numerator is circled, with an arrow pointing to a red 0 to its right.
- The term $1/u$ in the denominator is circled, with an arrow pointing to a red 0 below it.
- The term $7 \operatorname{sh}(u) / u^2$ in the denominator is circled, with an arrow pointing to a red 0 below it.