Mathematik 1

2012-01-30

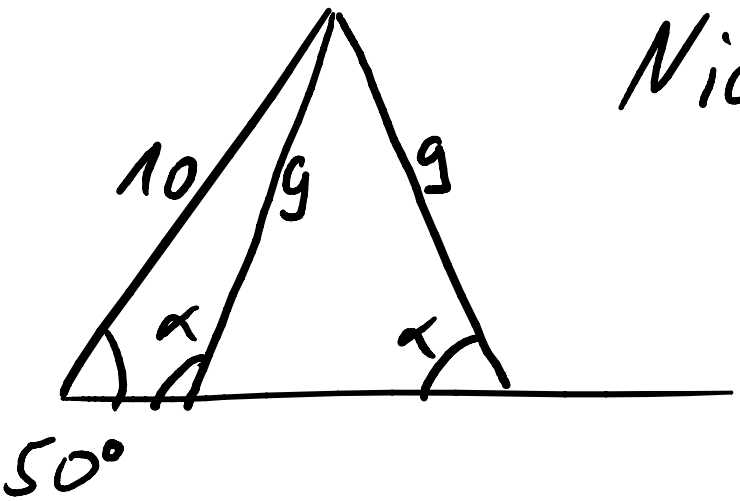
1) $\left(7+\log_3(x^2)\right)^3 = 1000$

$\Leftrightarrow 7+\log_3(x^2) = 10$

$\Leftrightarrow \log_3(x^2) = 3$

$\Leftrightarrow x^2 = 3^3 = 27 \Leftrightarrow x = \pm\sqrt{27}$

2)

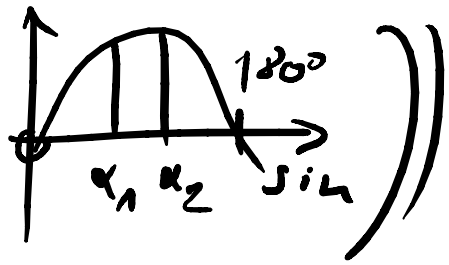


Nicht eindeutig!

$$\frac{\sin(\alpha)}{10} = \frac{\sin(50°)}{g}$$

$$\Rightarrow \sin(\alpha) = \frac{10}{g}\sin(50°)$$

$$\Rightarrow \begin{cases} \alpha_1 = \arcsin\left(\frac{10}{J}\sin(50°)\right) \\ \alpha_2 = 180° - \alpha_1 \end{cases}$$

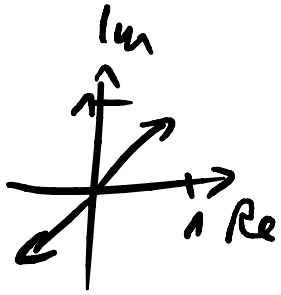
((

))

3) $z^2 - 2iz - 1 - i = 0$

$$\Leftrightarrow z = i \pm \sqrt{i^2 + 1 + i}$$

$$= i \pm \sqrt{i}$$



$$= i \pm \left(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right)$$

$$= \pm\frac{1}{\sqrt{2}} + \left(1 \pm \frac{1}{\sqrt{2}}\right)i \qquad \begin{matrix} ++ \\ \text{oder} \\ -- \end{matrix}$$

4) $$\frac{df}{dx} = \frac{\sqrt{x}\cos(3x)\cdot 3 - \frac{1}{2\sqrt{x}}\sin(3x)}{x}$$

((
$$= \frac{3\cos(3x)}{\sqrt{x}} - \frac{\sin(3x)}{2x^{3/2}}$$
))

5) $u = \sin(3x)$

$\Rightarrow \frac{du}{dx} = 3\cos(3x)$

$\Rightarrow$ „$du = 3\cos(3x)\,dx$“

Also Integral $= \int_{u(0)}^{u(5)} u^2 \frac{du}{3}$

$= \frac{1}{3}\int_0^{\sin(15)} u^2\,du = \frac{1}{3}\left[\frac{u^3}{3}\right]_0^{\sin(15)}$

$= \frac{1}{9}(\sin(15))^3$

6) $E[X] = \frac{1}{100}\cdot 0$

$+ \frac{99}{100}\cdot\frac{1}{6}\cdot 1$

$+ \dots$

$+ \frac{99}{100}\cdot\frac{1}{6}\cdot 6$

$= \frac{99}{100}\cdot 3\frac{1}{2}$ $((= 3{,}5 - 0{,}035 = 3{,}465))$

7) Verlauf?

$$x^3 - 400x^2 + 40000x$$

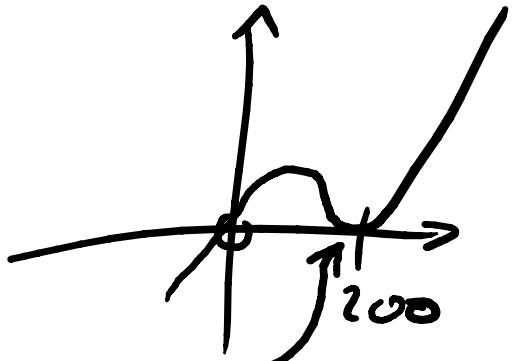
$$= x(\underbrace{x^2 - 400x + 40000}_{\text{Nullstellen?}})$$

$$x = 200 \pm \sqrt{40000 - 40000}$$

$$= 200$$

$$= x(x-200)^2$$

Also Verlauf:



Also positiv bei $x = 193{,}75$.

8) $f(x) = \frac{x}{2} + 1 + \frac{A}{x}$

Asymptote ✓

Polstelle ✓ ($A \neq 0$)

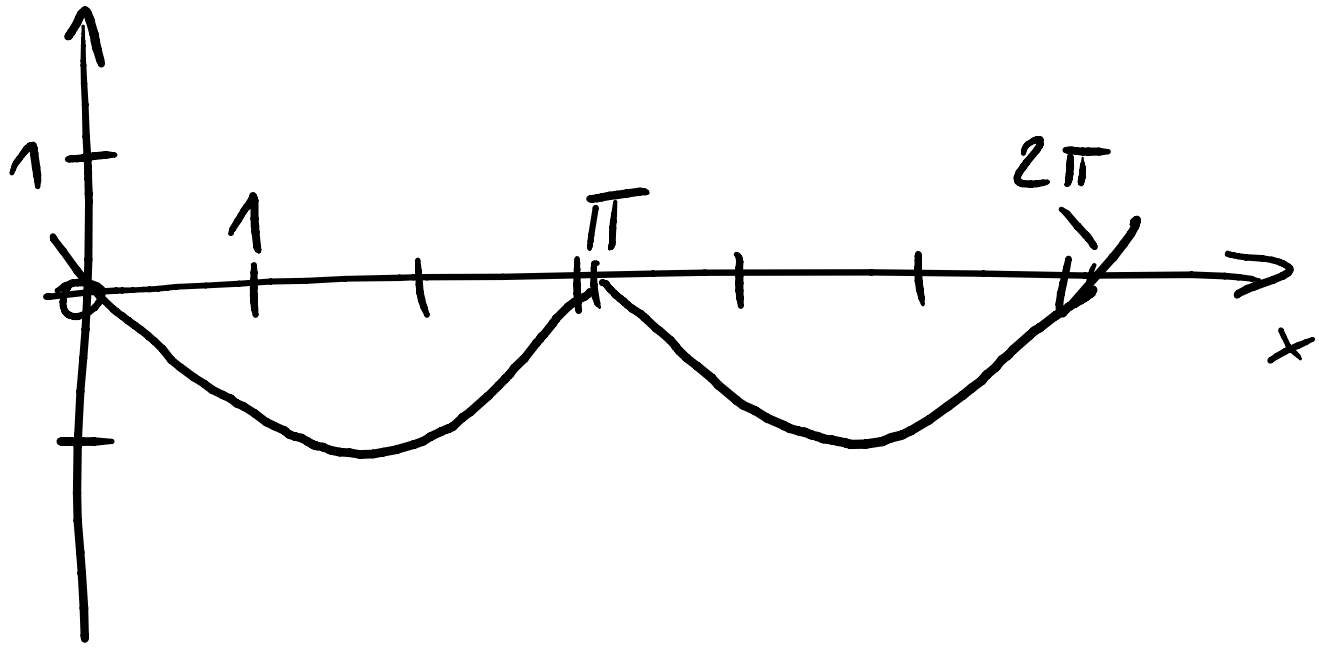
Nullstelle:

$$0 \overset{!}{=} \frac{1}{2} + 1 + \frac{A}{1}$$

$$\Leftrightarrow A = -\frac{3}{2}$$

((weitere mögliche Funktionen z.B. mit $f(x) = \frac{x}{2} + 1 + \frac{Bx+C}{x^2}$))

g) Für $x - \pi \geq 0$ kann man die Betragsstriche weglassen; links davon muss es spiegelsymmetrisch weitergehen.



10) $(x+1)^2 \leq x$

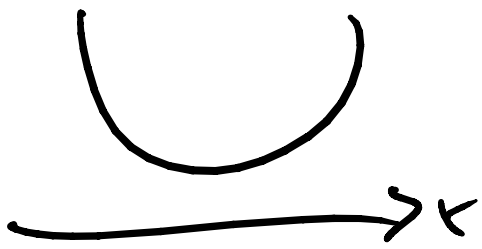
$\Leftrightarrow x^2 + 2x + 1 \leq x$

$\Leftrightarrow \underbrace{x^2 + x + 1}_{\text{Nullstellen?}} \leq 0$

$x = -\frac{1}{2} \pm \sqrt{\frac{1}{4} - 1}$ $\quad < 0$

Keine reellen Nullstellen!

Situation:



Also nie ≤ 0.

Also $\mathbb{L} = \{\}$.

11) Zahl aller Möglichkeiten:

$26 \cdot 25 \cdot 24 \cdot 23 \cdot 22 \cdot 21$

Zahl der günstigen Mögl.:

CODE ist: $22 \cdot 21$

* CODE : "

** CODE : "

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zusammen $3 \cdot 22 \cdot 21$

Wahrscheinlichkeit

$$= \frac{3 \cdot 22 \cdot 21}{26 \cdot 25 \cdot 24 \cdot 23 \cdot 22 \cdot 21} \cdot 8$$

12) $$\frac{n \cos(n!) - 5n^2}{\sqrt{3n^4 + 7}} = \frac{\frac{1}{n}\cos(n!) - 5}{\sqrt{\frac{3n^4+7}{n^4}}}$$

$$= \frac{\frac{1}{n}\cos(n!) - 5}{\sqrt{3 + \frac{7}{n^4}}}$$

$\frac{1}{n}\cos(n!) \to 0$, $-5 \to -5$, $\sqrt{3 + \frac{7}{n^4}} \to \sqrt{3}$

$\to -5/\sqrt{3}$