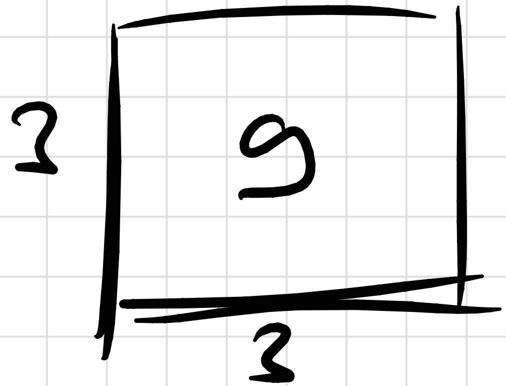
Potenzen

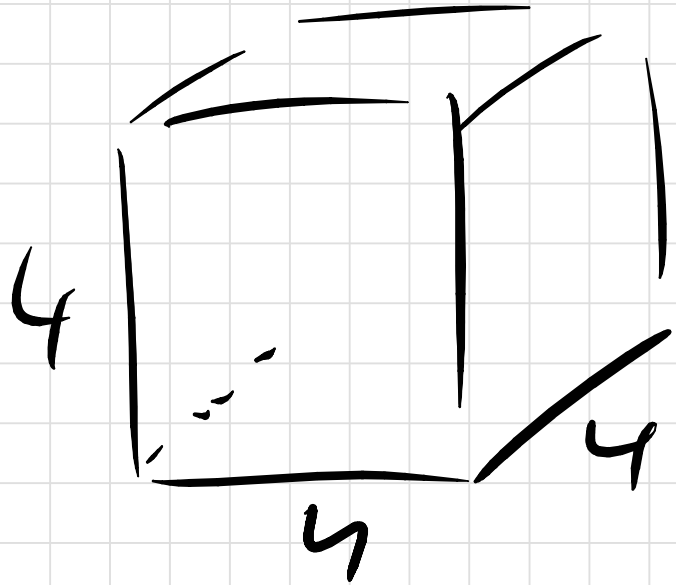
Exponent

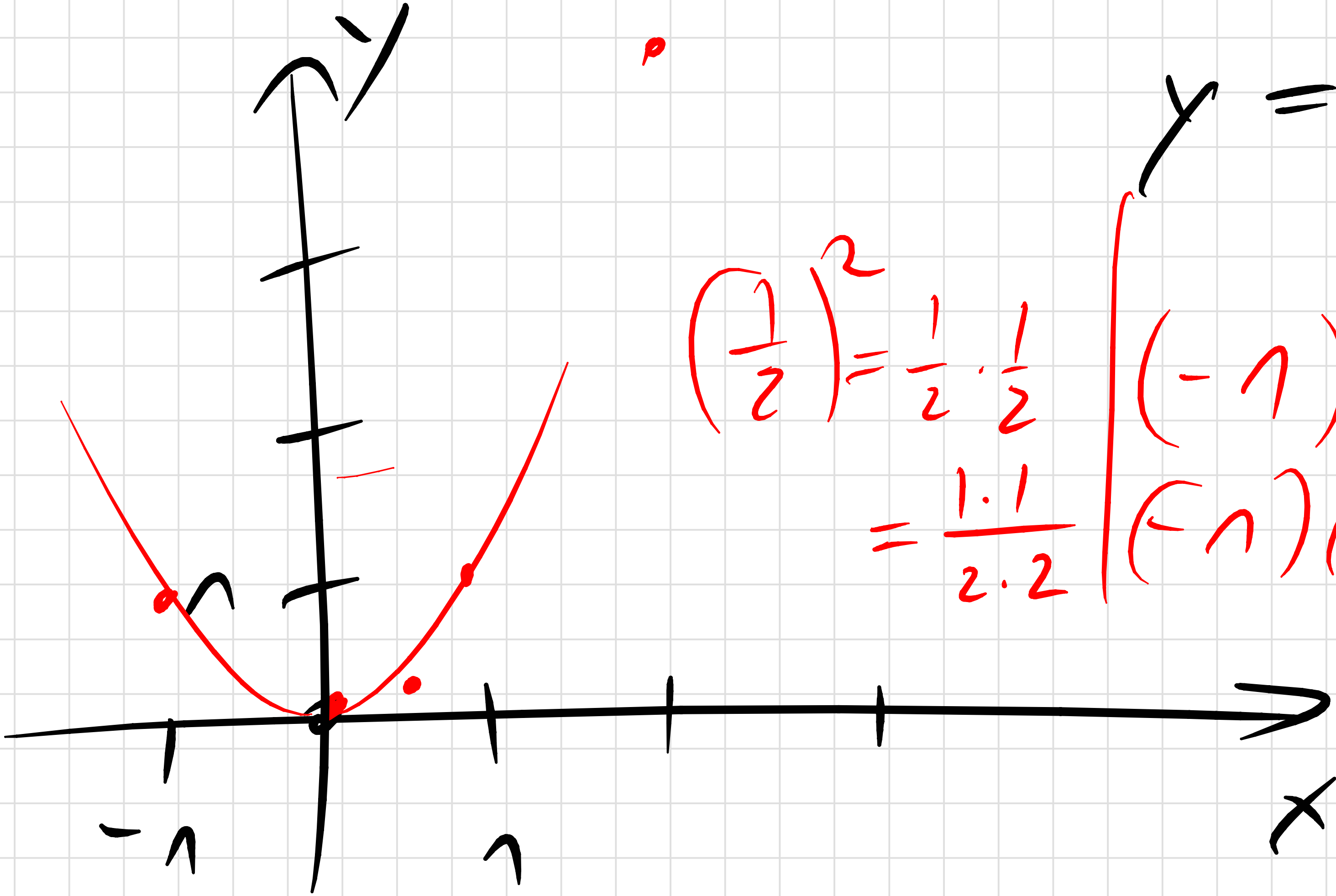
$$3^2 = 3 \cdot 3 = 9$$

Basis



$$4^3 = 4 \cdot 4 \cdot 4 = 64$$

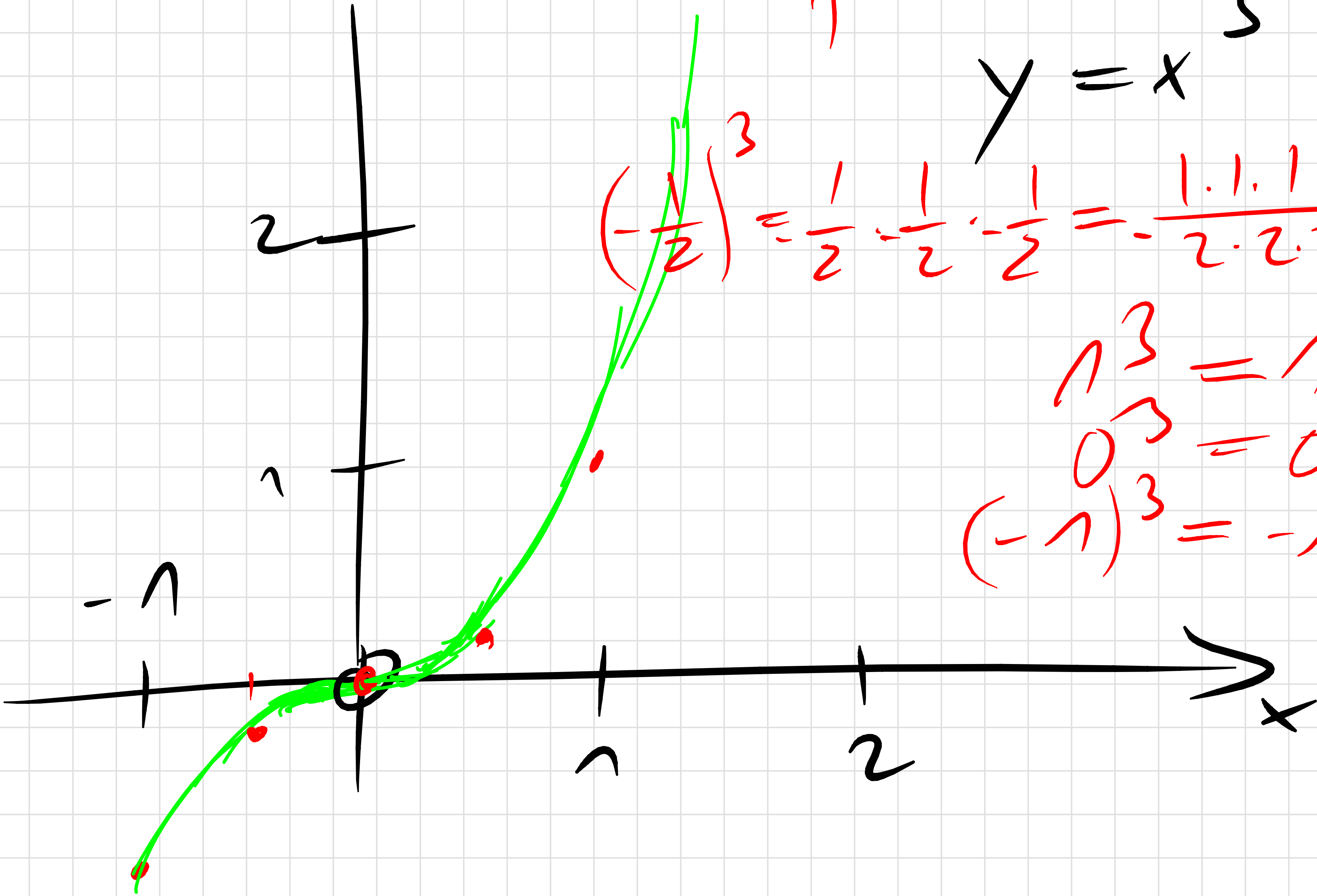




$$y = x^2$$

$$\left(\frac{1}{2}\right)^2 = \frac{1}{2}\cdot\frac{1}{2} = \frac{1\cdot 1}{2\cdot 2}$$

$$(-1)^2 = (-1)(-1) = 1$$



$$y = x^3$$

8

$$\left(-\frac{1}{2}\right)^3 = -\frac{1}{2}\cdot -\frac{1}{2}\cdot -\frac{1}{2} = -\frac{1\cdot 1\cdot 1}{2\cdot 2\cdot 2}$$

$$1^3 = 1$$

$$0^3 = 0$$

$$(-1)^3 = -1$$

$$a^7 = a \cdot a \cdot a \cdot a \cdot a \cdot a \cdot a$$

$$(3 \cdot 4)^5 = (3 \cdot 4) \cdot (3 \cdot 4) \cdot (3 \cdot 4) \cdot (3 \cdot 4) \cdot (3 \cdot 4)$$

$$= 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$$

$$= 3^5 \cdot 4^5$$

$$\left(\frac{3}{4}\right)^5 = \frac{3}{4} \cdot \frac{3}{4} \cdot \frac{3}{4} \cdot \frac{3}{4} \cdot \frac{3}{4}$$

$$= \frac{3^5}{4^5}$$

$$3^{4+5} = 3^9 = \underbrace{3\cdot 3\cdot 3\cdot 3}_{3^4}\cdot\underbrace{3\cdot 3\cdot 3\cdot 3\cdot 3}_{3^5}$$

$$3^{7-2} = 3^5 = 3\cdot 3\cdot 3\cdot 3\cdot 3\cdot\frac{3\cdot 3}{3\cdot 3}$$

$$= \frac{3^7}{3^2}$$

$$3^1 = ?$$

$$3^1 \cdot 3^5 = 3^6$$

wenn Rechenregeln te

$$\Rightarrow 3^1 = \frac{3^6}{3^5} = \frac{\cancel{3}\cdot\cancel{3}\cdot\cancel{3}\cdot\cancel{3}\cdot\cancel{3}\cdot 3}{\cancel{3}\cdot\cancel{3}\cdot\cancel{3}\cdot\cancel{3}\cancel{3}} = 3$$

$3^0 = ?$

Rechenregel.

$$\underline{\underline{3^0 \cdot 3^4}} = 3^{0+4} = \underline{\underline{3^4}}$$

Muss 1 sein!

$$0^1 = 0$$

$$0^0 = \boxed{1}$$

$$3x^2 + 4x + 5 \cdot 1 = 3x^2 + 4x^1 + 5x^0$$

$0^3 = 0 \, , \; 0^2 = 0 \, , \; 0^1 = 0 \, , \; 0^0 = 1$

$3^{42}, \quad 3^{2}, \quad 3^{1}, \quad 3^{0} \quad \ldots$

$$\boxed{3^{-4} = ?}$$

$\dfrac{3^{5}}{3^{9}} = 3^{-4}$ wenn Rechenregeln da

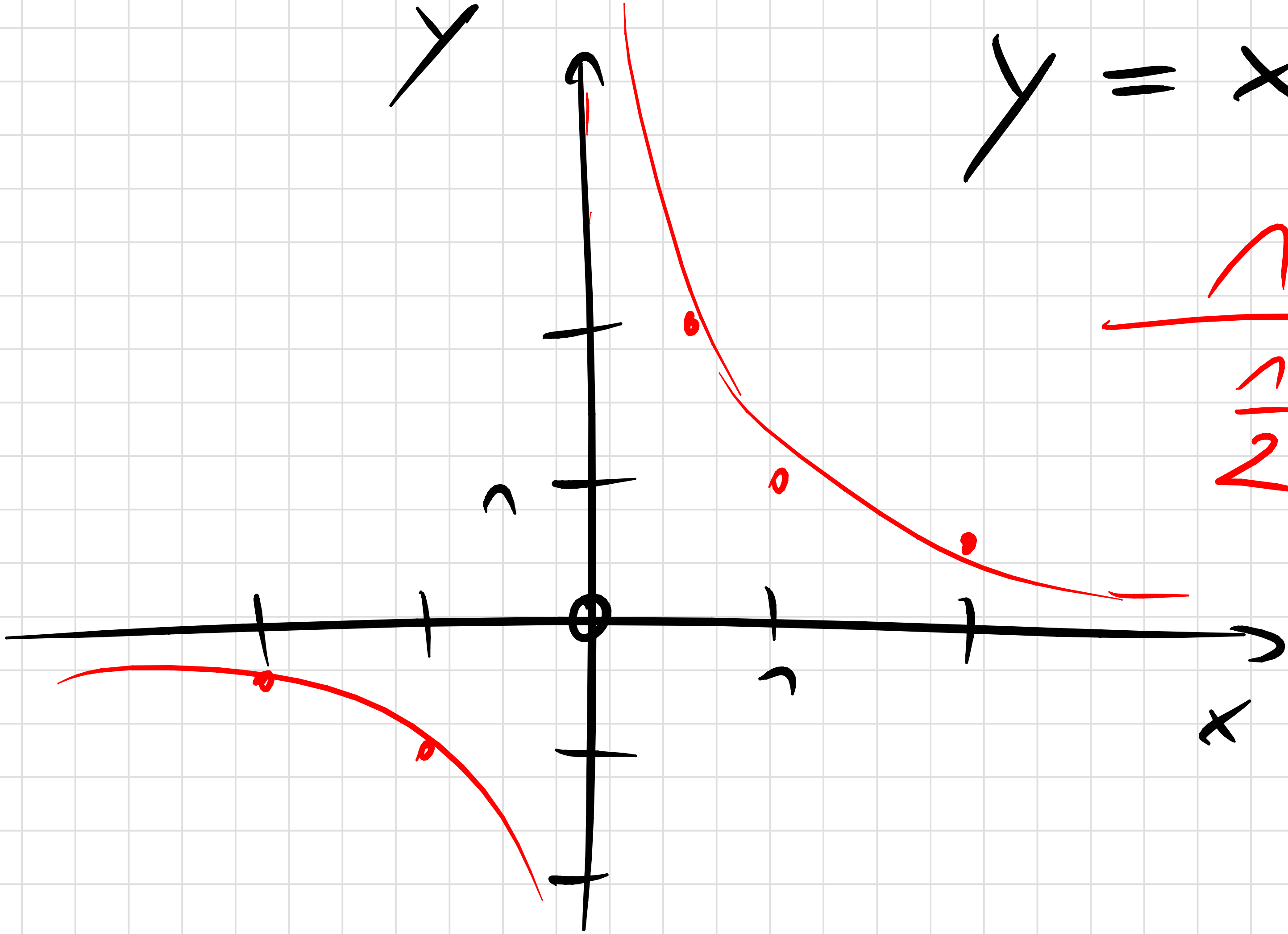
$$\frac{\cancel{3\cdot 3\cdot 3\cdot 3\cdot 3}}{3\cdot 3\cdot 3\cdot 3\cdot \cancel{3\cdot 3\cdot 3\cdot 3\cdot 3}} = \frac{1}{3^4}$$

$$x^{-n} = \frac{1}{x^n}$$

für $x \neq 0$

und $n = 1, 2, 3, \ldots$

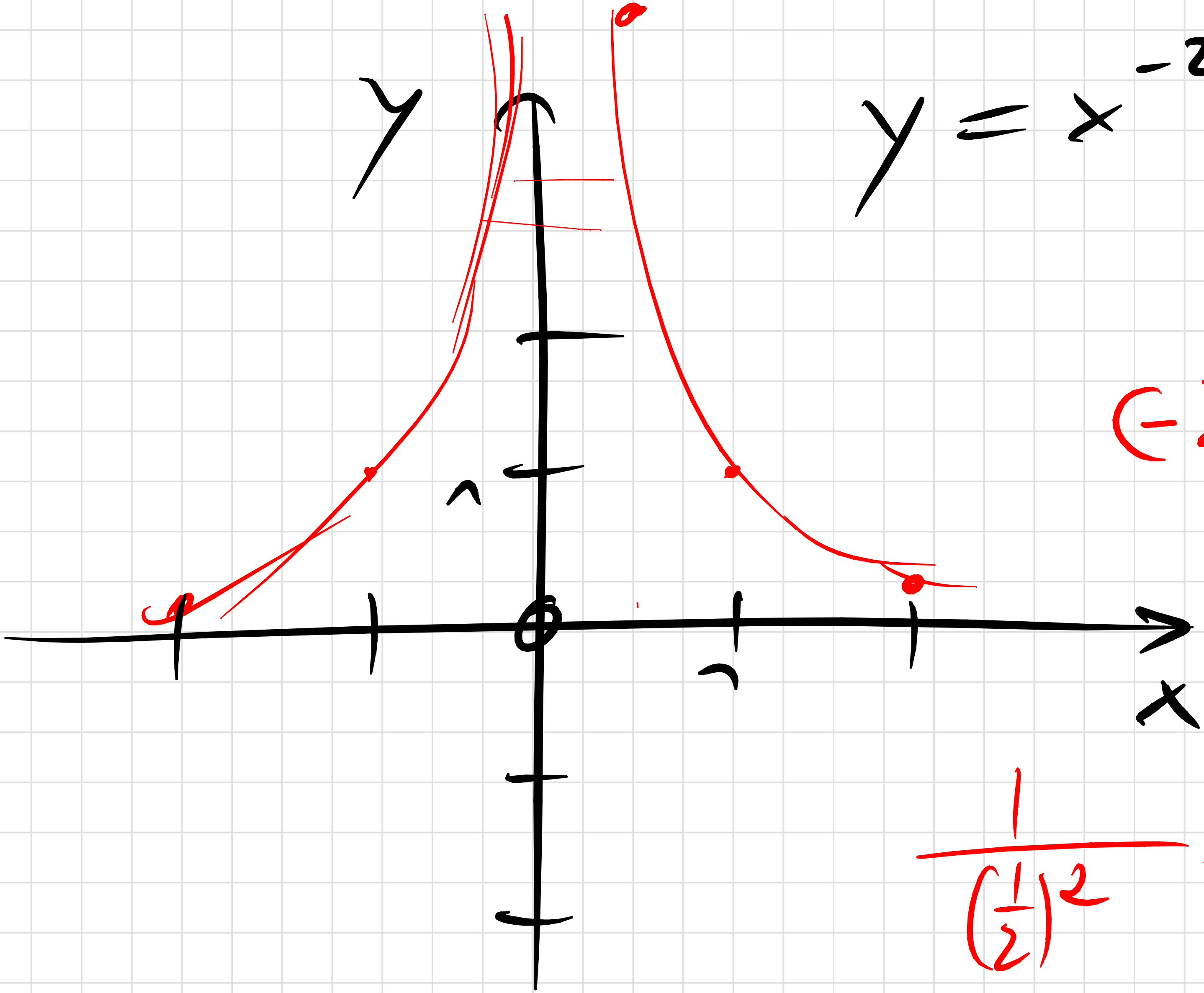
$$y = x^{-1} = \frac{1}{x}$$



$$\frac{1}{\frac{1}{2}} = 1 \cdot \frac{2}{1}$$

$$\frac{1}{-2} = -\frac{1}{2}$$

$$y = x^{-2} = \frac{1}{x^2}$$



$$\frac{1}{(-2)^2} = \frac{1}{4}$$

$$\frac{1}{1^2}$$

$$\frac{1}{\left(\frac{1}{2}\right)^2} = \frac{1}{\frac{1}{4}} = 1 \cdot \frac{4}{1}$$

$3^{1/2} = ?$

$$3^{2\cdot 5} = 3^{10} = \overbrace{3 \cdot \ldots \cdot 3}^{10 \text{ Faktoren}}$$

$$= \underbrace{(3\cdot 3) \cdot \ldots \cdot (3\cdot 3)}_{5 \text{ Faktoren}}$$

$$= \left(3^2\right)^5$$

Wenn Rechenregel weiter gilt:

$$\left(3^{1/2}\right)^2 = 3^{\frac{1}{2}\cdot 2} = 3^1 = 3$$

Also wähle $3^{1/2} = \sqrt{3}$.

$3^{1/5} = ?$

$$\left(3^{\cancel{1/5}}\right)^5 = 3^{\frac{1}{5}\cdot 5} = 3^1 = 3$$

Wähle $3^{1/5} = \sqrt[5]{3}$

$$\left(3^{7/5}\right)^5 = 3^{\frac{7}{\cancel{5}}\cdot\cancel{5}} = 3^7$$

Wähle $3^{7/5} = \sqrt[5]{3^7}$

$$3^{-7/5} = \frac{1}{3^{7/5}}$$

$$= \frac{1}{\sqrt[5]{3^7}}$$

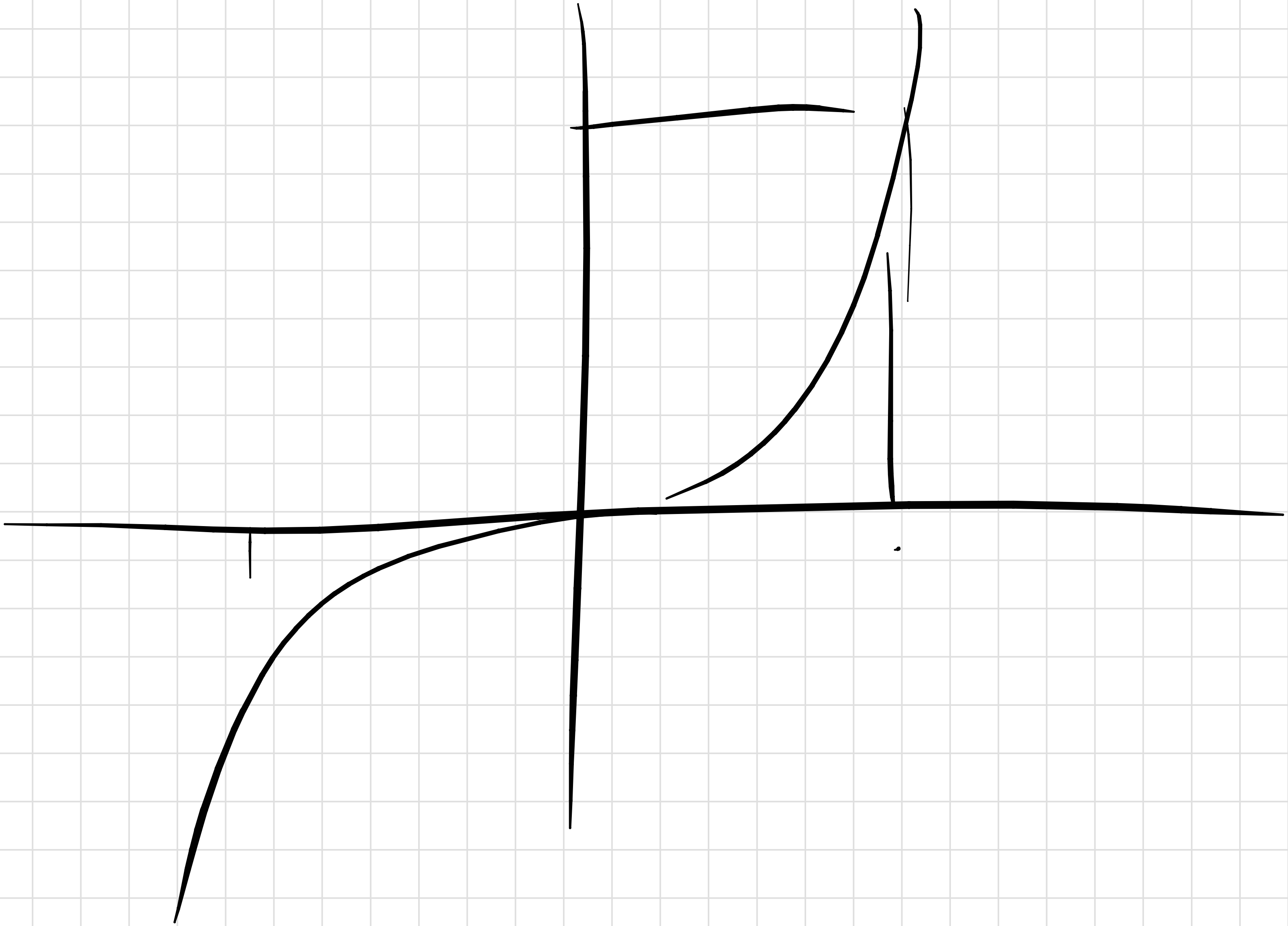
Wurzeln

$$x^5 = 3 \iff x = \sqrt[5]{3}$$

$$x^2 = 49 \iff x = \sqrt{49} = 7$$

$$\vee \; x = -\sqrt{49} = -7$$

$$\sqrt[3]{-8} = -2 \; ?$$



$$\sqrt[3]{\frac{27}{64}} = \frac{\sqrt[3]{27}}{\sqrt[3]{64}} = \frac{3}{4}$$

$$\sqrt[3]{27 \cdot 64} = \sqrt[3]{27} \cdot \sqrt[3]{64} = 3 \cdot 4$$

$$\sqrt[3]{2+5} = ?$$

$$1{,}2356 \cdot \underbrace{10^{20}}$$

$$\underbrace{10 \cdot 10 \cdot \ldots \cdot 10}_{20x}$$

$$= 1\underbrace{0 \ldots 0}_{20 \text{ Nullen}}$$

$10^1 = 10$
$10^2 = 100$
$10^3 = 1000$

10^{100} = Googol

10^{Googol} = Googolplex

$$1{,}2340 \cdot 10^{13}$$

$$= 1\,2\,3\,4\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0$$

signifikant

$0{,}000000000123450$

$1 \quad 2 \quad 3 \quad 4 \qquad\qquad\qquad 10$

$= 1{,}23450 \cdot 10^{-10}$

Potenzfunktion

$$y = x^n$$

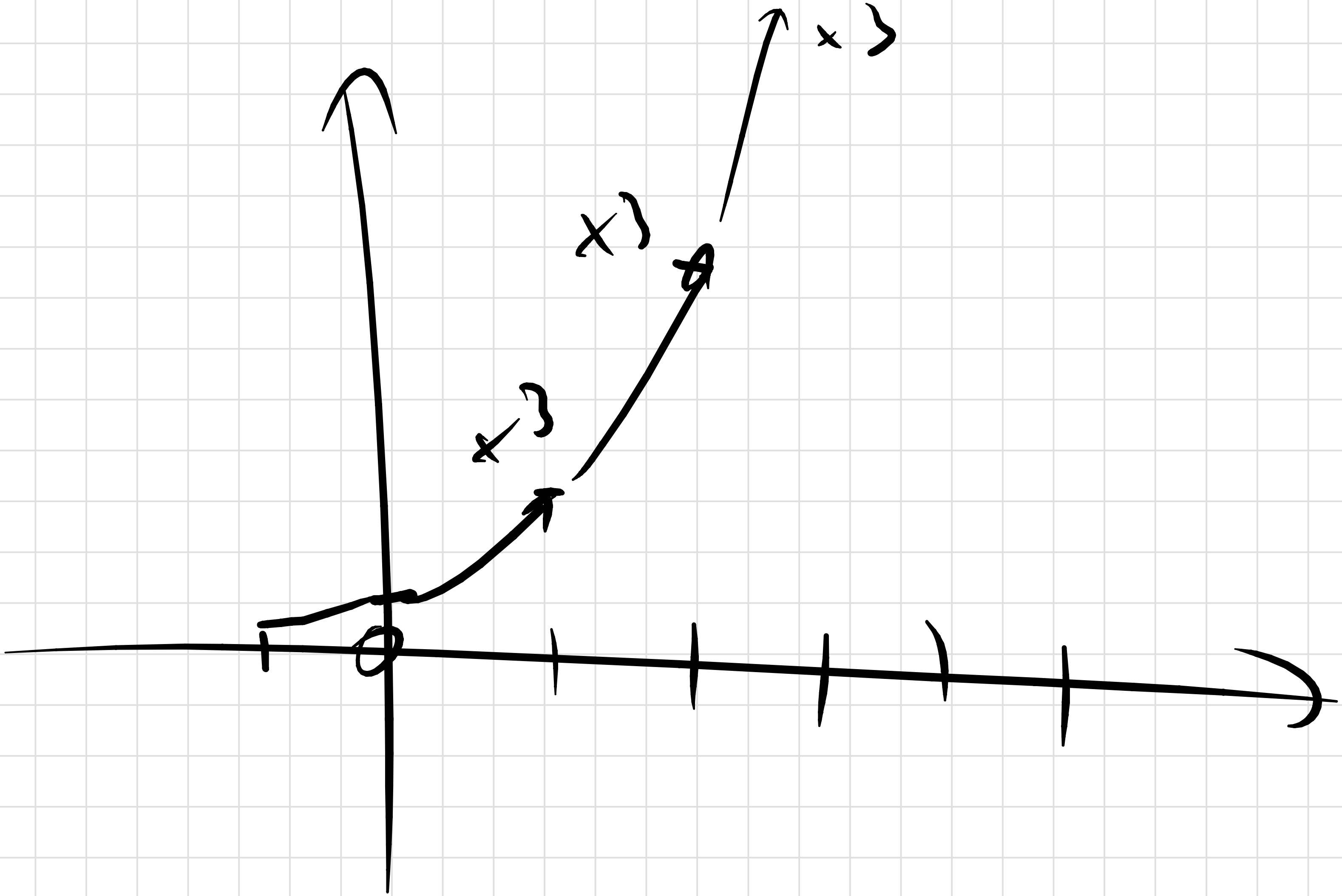
Exponentialfunktion

$$y = a^x$$

$$3^{-1} \cdot 3^{1} = 3^{-1+1} = 3^{0}$$

$$= 1$$

$$\Rightarrow 3^{-1} = \frac{1}{3}$$



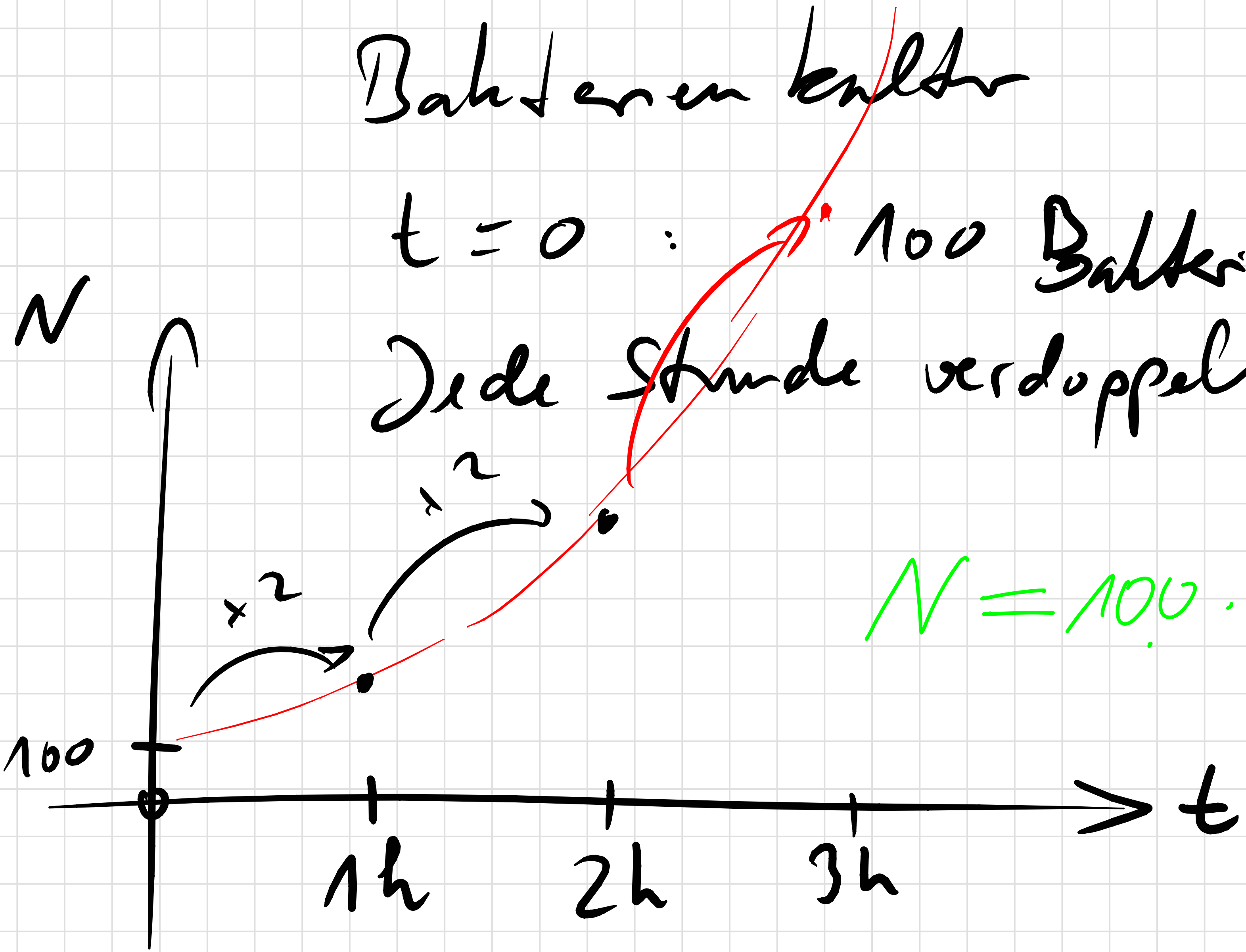
0
x3
x3
x3

Exponentielles Wachstum

Bakterienkultur

$t = 0$: 100 Bakterien

Jede Stunde verdoppeln!

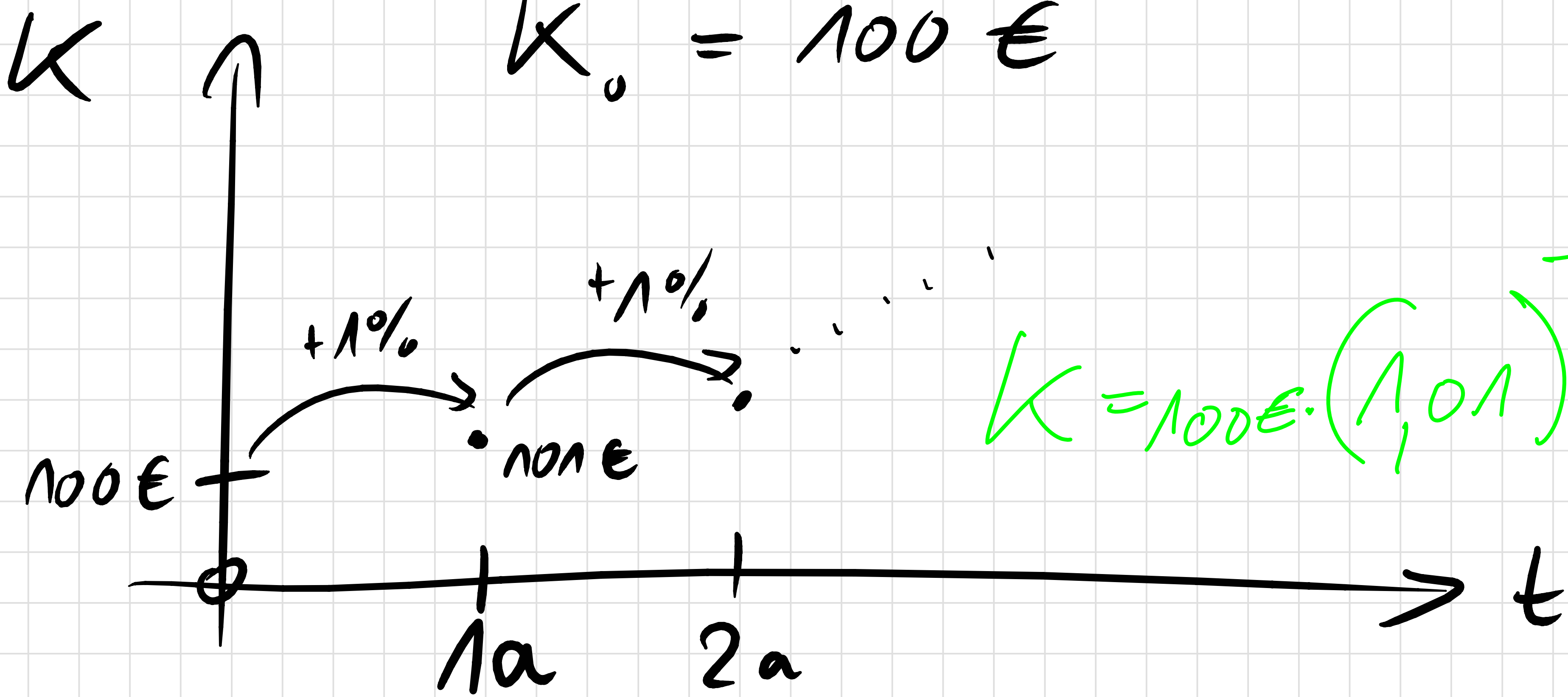


$N = 100 \cdot 2^{t/1h}$

Zinseszins

1% p.a.

$K_0 = 100\,€$

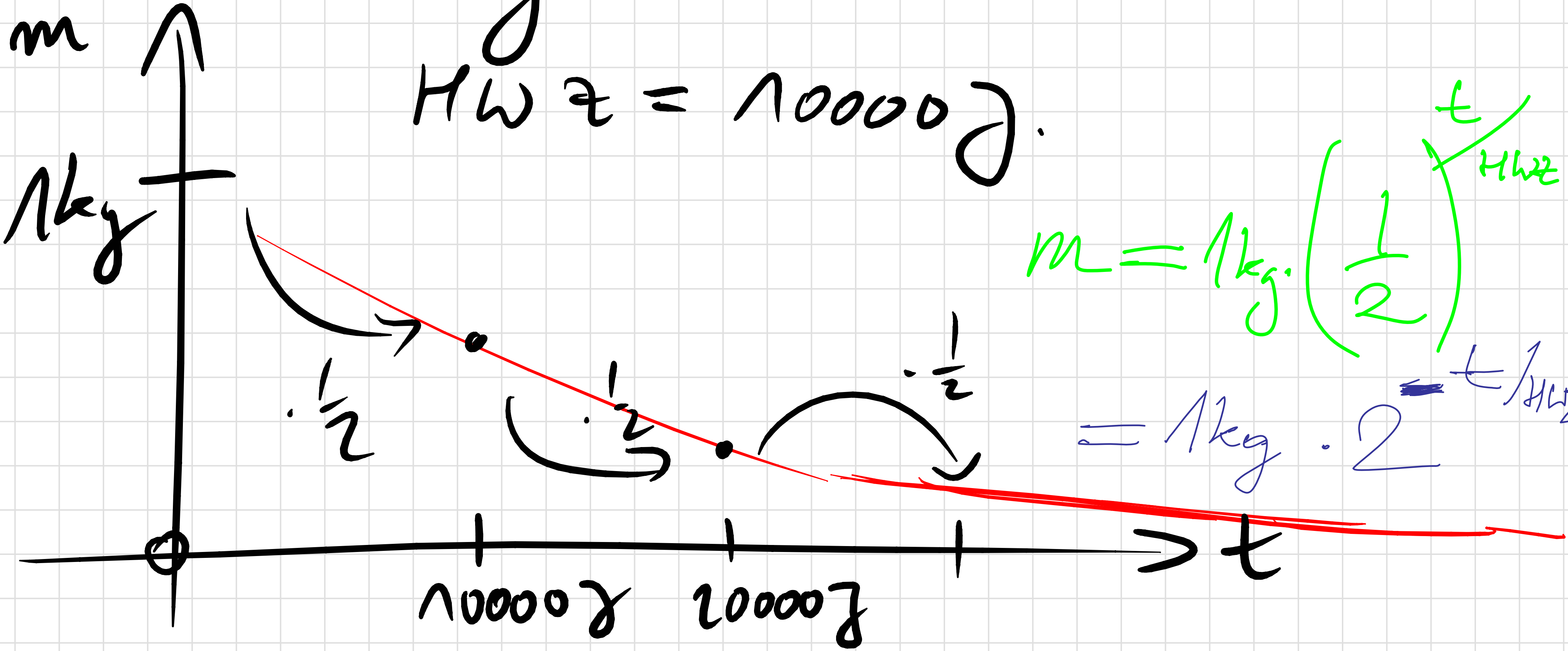


$K = 100\,€ \cdot (1{,}01)^{t/1a}$

Exponentieller Zerfall

1kg radioaktives Material

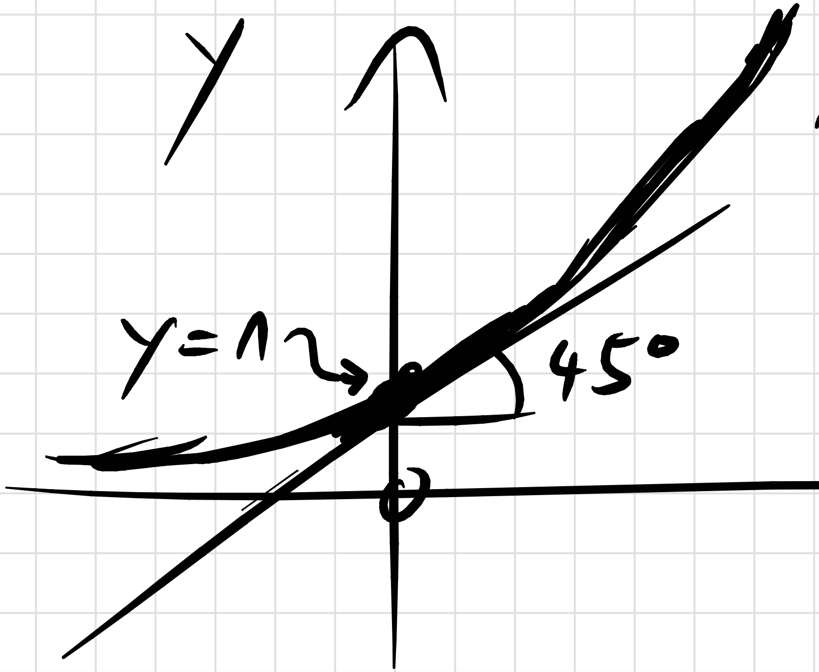
HWZ = 10000 J.



$$m = 1kg \cdot \left(\frac{1}{2}\right)^{t/HWZ}$$

$$= 1kg \cdot 2^{-t/HWZ}$$

Die ~~„natürliche"~~ Exponentialfunktion



$y = \exp(x)$

$= \underbrace{2{,}718\ldots}^{x}$

e = Eulersche Zahl

Logarithmen

dekadische Logarithmus

$$10^x = 10000 \quad \Big| \; \lg = \log_{10}$$

$$\Leftrightarrow \quad x = 4$$

$$\lg(0{,}01) = -2$$

dem $10^{-2} = 0{,}01$

$$\log_3\left(\frac{1}{9}\right) = -2$$

$$3^{-2} = \frac{1}{3^2} = \frac{1}{9}$$

$$\cancel{3^{-3} = \frac{1}{3^3} = \frac{1}{27}}$$

$\log_3(1) = 0$

donc $3^0 = 1$

>0

$$\log_{\bullet}\left(\bullet\right)$$

>0, aber nicht 1

$$\boxed{\begin{aligned} \log_{10} &= \lg \\ \log_e &= \ln \end{aligned}}$$

$$\log_2 = \mathrm{lb} = \mathrm{ld}$$

$$\cancel{\log_a(x+y)} =$$

$x, y > 0, \quad n > 0$

$$\log_a(x \cdot y) = \log_a(x) + \log_a(y)$$

$$\log_a(x/y) = \log_a(x) - \log_a(y)$$

$$\log_a(x^n) = n \log_a(x)$$

$$\log_a(\sqrt[n]{x}) = \frac{\log_a(x)}{n}$$

$$\log_3\Big(\underset{3^3}{27}\cdot\underset{3^2}{9}\Big)=\log_3\Big(\underset{3^5}{243}\Big)=5$$

$$=\log_3\left(3^3\right)+\log_3\left(3^2\right)=3+2$$

$$7^x = 13$$

$$\Leftrightarrow x = \log_7(13)$$

Anders: $7 = 10^{\lg(7)}$

$$\Rightarrow 13 = 7^x = \left(10^{\lg(7)}\right)^x$$

$$= 10^{\lg(7) \cdot x}$$

$$\Leftrightarrow 13 = 10^{\boxed{\lg(7) \cdot x}\, bla}$$

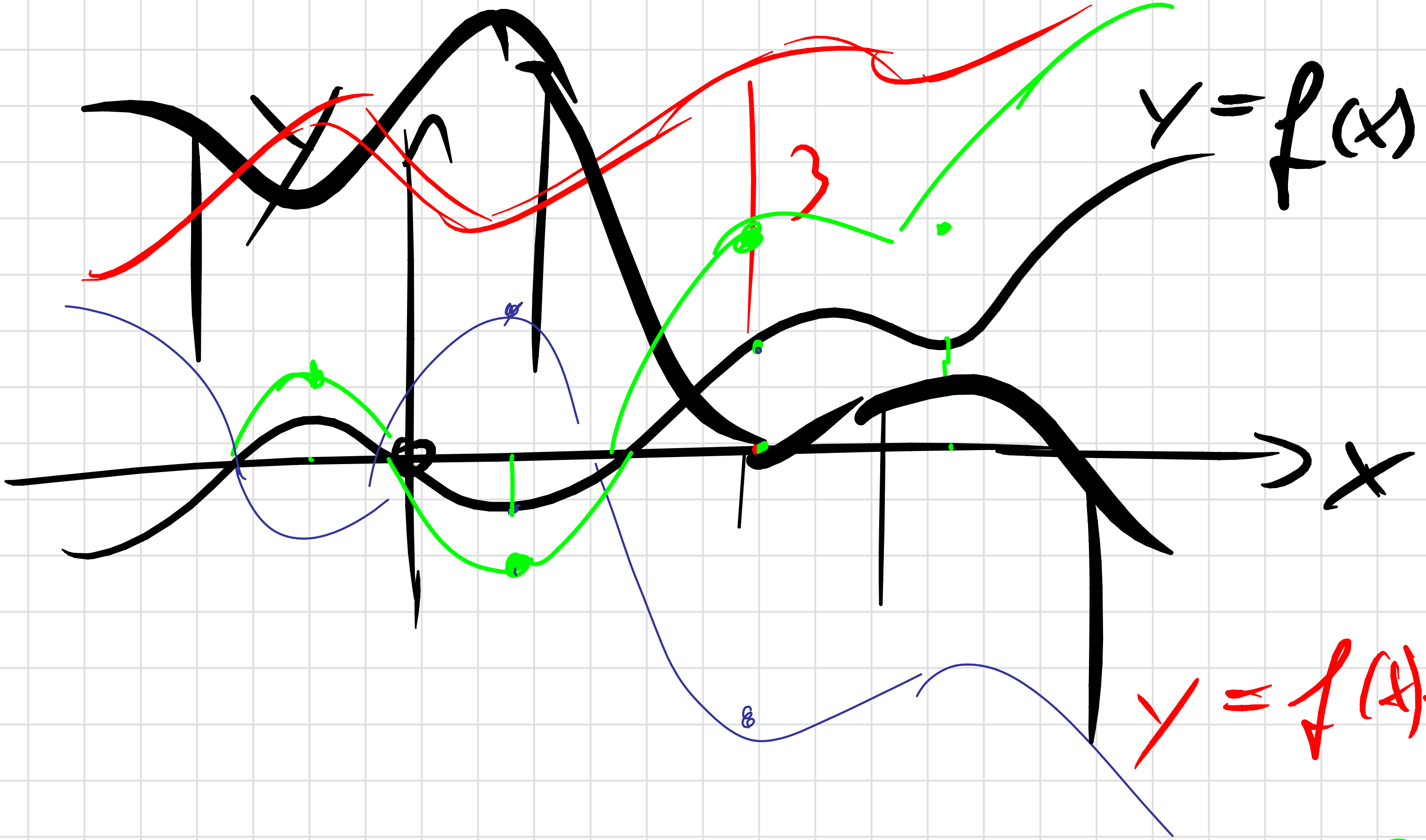
$$\Leftrightarrow \underbrace{\boxed{\lg(7) \cdot x}}_{bla} = \lg(13)$$

$$10^{bla} = 13 \qquad \lg(13)$$

$$\Leftrightarrow x = \frac{\lg(13)}{\lg(7)}$$

Verkettete Funktionen

$$5 \cdot f(3x + 4) + 6$$



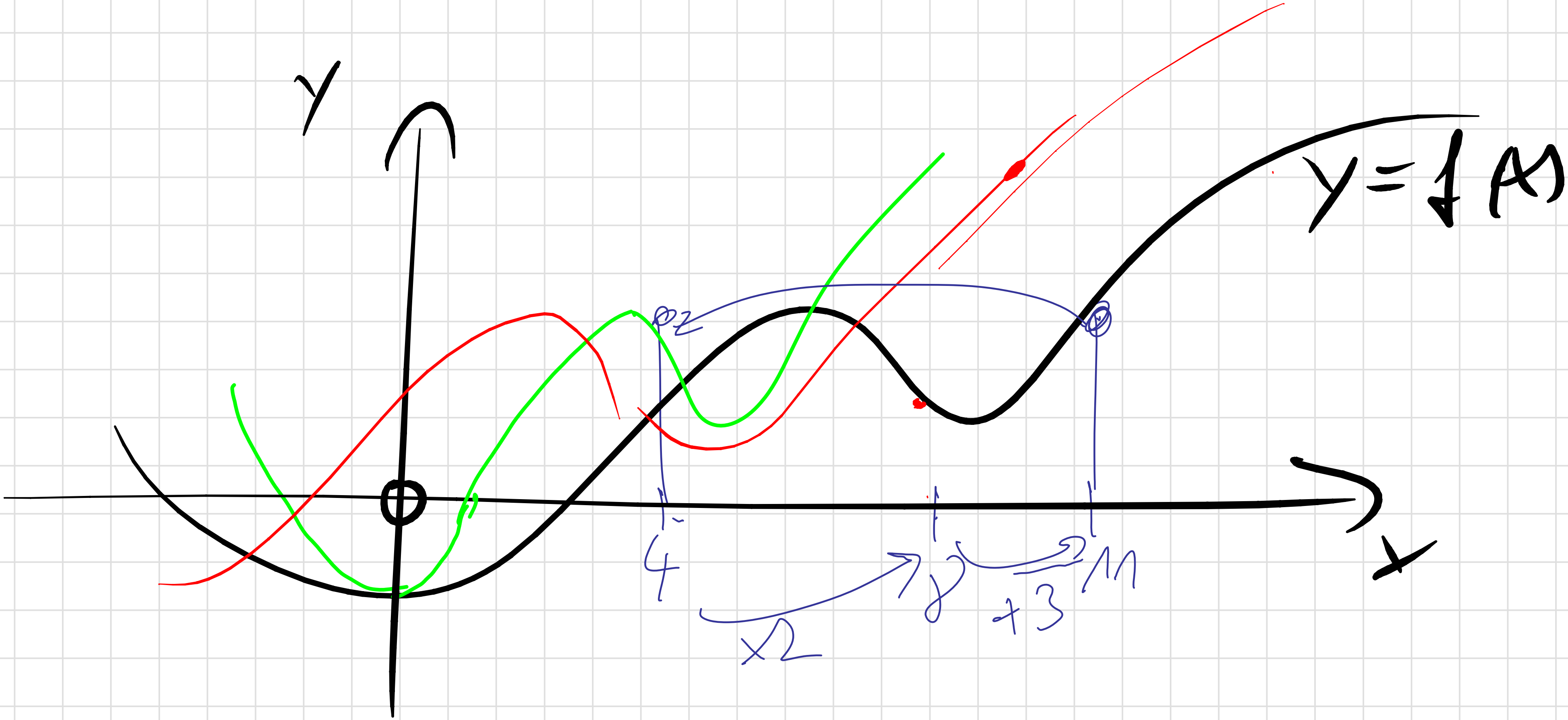
$y = f(x)$

$y = f(x) + 3$

$y = 2f(x)$

$y = -2f(x)$

$y = -2f(x) + 3$



$y = f(2x)$

$y = f(x+3)$

um 3 nach **links**

um drei nach links und dann auf $\frac{1}{2}$ stauchen

$y = f(2x+3)$