1) $$\underbrace{\log_2 \sqrt{3^{-x}+1}}_{\frac{1}{2}\log_2\left(3^{-x}+1\right)} = 3$$

$$\iff \log_2\left(3^{-x}+1\right) = 6$$

$$\iff 3^{-x}+1 = 2^6 = 64$$

$$\iff 3^{-x} = 63 \iff x = -\log_3(63).$$

2) $$\frac{x-1}{x-2} \le 1 \iff x-2>0 \wedge x-1 \le x-2$$

$$\vee\ x-2<0 \wedge x-1 \ge x-2$$

$$\iff x>2 \wedge \overbrace{-1 \le -2}^{\text{falsch}}$$

$$\vee\ x<2 \wedge \underbrace{-1 \ge -2}_{\text{wahr}}$$

$$\iff x<2$$

d.h. $\mathbb{L} = (-\infty; 2)$

3) Cosinussatz. Eindeutig.

$$\text{Winkel} = \arccos \underbrace{\frac{2^2+6^2-5^2}{2\cdot 2\cdot 6}}_{15/24\ =\ 5/8}$$

$$= \arccos(5/8)$$

4) $z^2 + iz + 3 = 0$

$$\Longleftrightarrow z = -\frac{i}{2} \pm \sqrt{\underbrace{\frac{-1}{4} - 3}_{-13/4}}$$

$$= -\frac{i}{2} \pm \frac{\sqrt{13}}{2} i$$

$$= 0 + \frac{1}{2}\left(-1 \pm \sqrt{13}\right) i$$

5) $\frac{d}{dx}\left(\frac{\sin(3x)}{x^2+1}\right) =$

$$\frac{(x^2+1)\cos(3x)\cdot 3 - 2x\sin(3x)}{(x^2+1)^2}$$

6) Bestimmung von C:

$$1 \overset{!}{=} \int_1^3 c\cdot(x-1)\,dx = c\int_1^3 (x-1)\,dx$$

$$= c\cdot\left[\frac{x^2}{2} - x\right]_1^3 = c\cdot\underbrace{\left(\frac{9}{2} - 3 - \frac{1}{2} + 1\right)}_{2}$$

$$\Longrightarrow c = 1/2$$

(Hätte man auch geometrisch machen können.)

$$P[\{X \in [2;3]\}] = \int_2^3 \frac{1}{2}(x-1)\,dx = \frac{\left[\frac{x^2}{2} - x\right]_2^3}{2}$$

$= \frac{1}{2}\left(\frac{9}{2} - 3 - \frac{4}{2} + 2\right) = 3/4$

7) Einsetzen: $x^4 + x^4 = 1 \iff x = \pm\frac{1}{\sqrt[4]{2}}$

Also zwei Punkte:

$\left(\frac{1}{\sqrt[4]{2}}, \frac{1}{\sqrt[4]{2}}\right)$ und $\left(-\frac{1}{\sqrt[4]{2}}, -\frac{1}{\sqrt[4]{2}}\right)$

8) $N_0 \cdot 2^{-300a/HWZ} = \frac{N_0}{10}$ ((a = annum))

$\Rightarrow 2^{-300a/HWZ} = 1/10$

$\Rightarrow -\frac{300a}{HWZ} = \log_2(1/10) = -\log_2(10)$

$\Rightarrow HWZ = \frac{300a}{\log_2(10)}$

9) A platzieren: 5 Möglichkeiten
B " : 4 "
C " : 3 "
D auf die beiden übrigen Plätze.

Also insgesamt $5 \cdot 4 \cdot 3 = 60$ Möglichkeiten.

10) Nenner muss den Grad 1 haben, also die Form $ax + b$ haben.

$$x^2 : (ax+b) = x + 2 \text{ Rest} \ldots$$

a muss 1 sein.

$$\frac{-(x^2+bx)}{-bx}$$

b muss 2 sein.

Also Nenner = $x-2$.

11) $$\frac{\sqrt{1+3n^2}}{4n+5\sin(n)} = \frac{\cancel{n}\sqrt{\frac{1}{n^2}+3}}{\cancel{n}\left(4+\frac{5}{n}\sin(n)\right)} \to \frac{\sqrt{3}}{4}$$

$\sqrt{3}$, 4

12) $$\text{Länge} = \int_1^2 \sqrt{\underbrace{1+\left(\frac{3}{2}x^{1/2}\right)}_{1+\frac{9}{4}x}}\,dx$$

$$u := 1+\frac{9}{4}x, \quad du = \frac{9}{4}dx$$

$$= \int_{1+\frac{9}{4}\cdot 1}^{1+\frac{9}{4}\cdot 2} \sqrt{u}\,\frac{4}{9}\,du = \frac{4}{9}\left[\frac{2}{3}u^{3/2}\right]_{13/4}^{22/4}$$

$$= \frac{8}{27}\left(\left(\frac{22}{4}\right)^{3/2} - \left(\frac{13}{4}\right)^{3/2}\right)$$

$$\left(\left(= \frac{\cancel{8}}{27\cdot\cancel{8}}\left(22^{3/2} - 13^{3/2}\right)\right)\right)$$