Verkettung von Funktionen

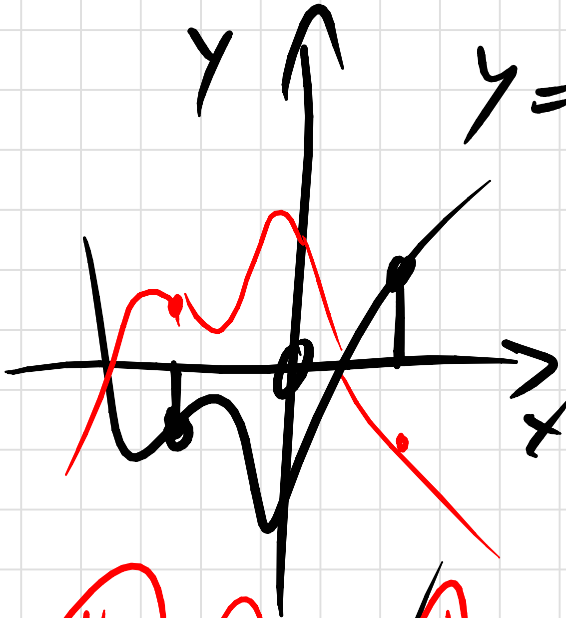
$$f(g(x))$$

$$\sin(e^x) \quad , \quad \sqrt{98+x^3}$$

---

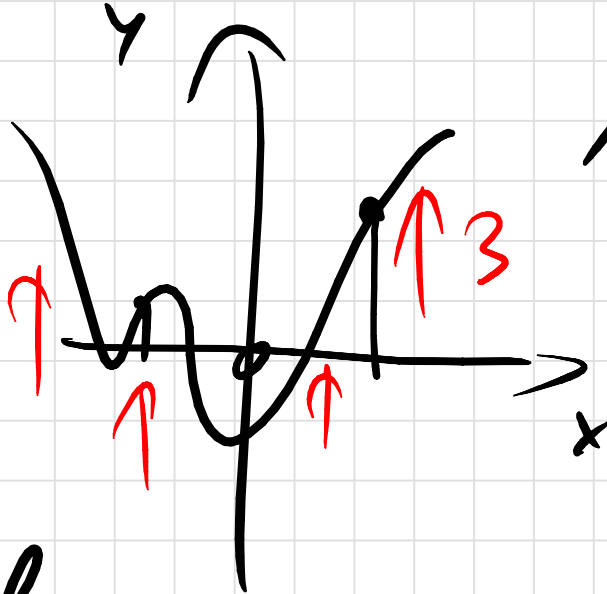
heute: $a f(b(x-c)) + d$

z.B. $42 \sin(3(x+4)) + 13$

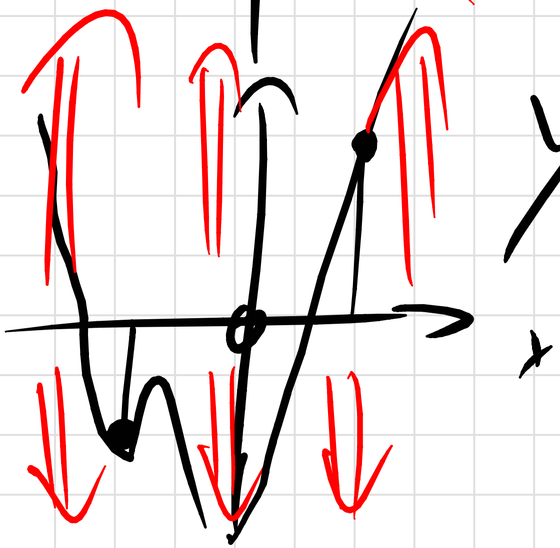


$y=f(x)$

$y=-f(x)$



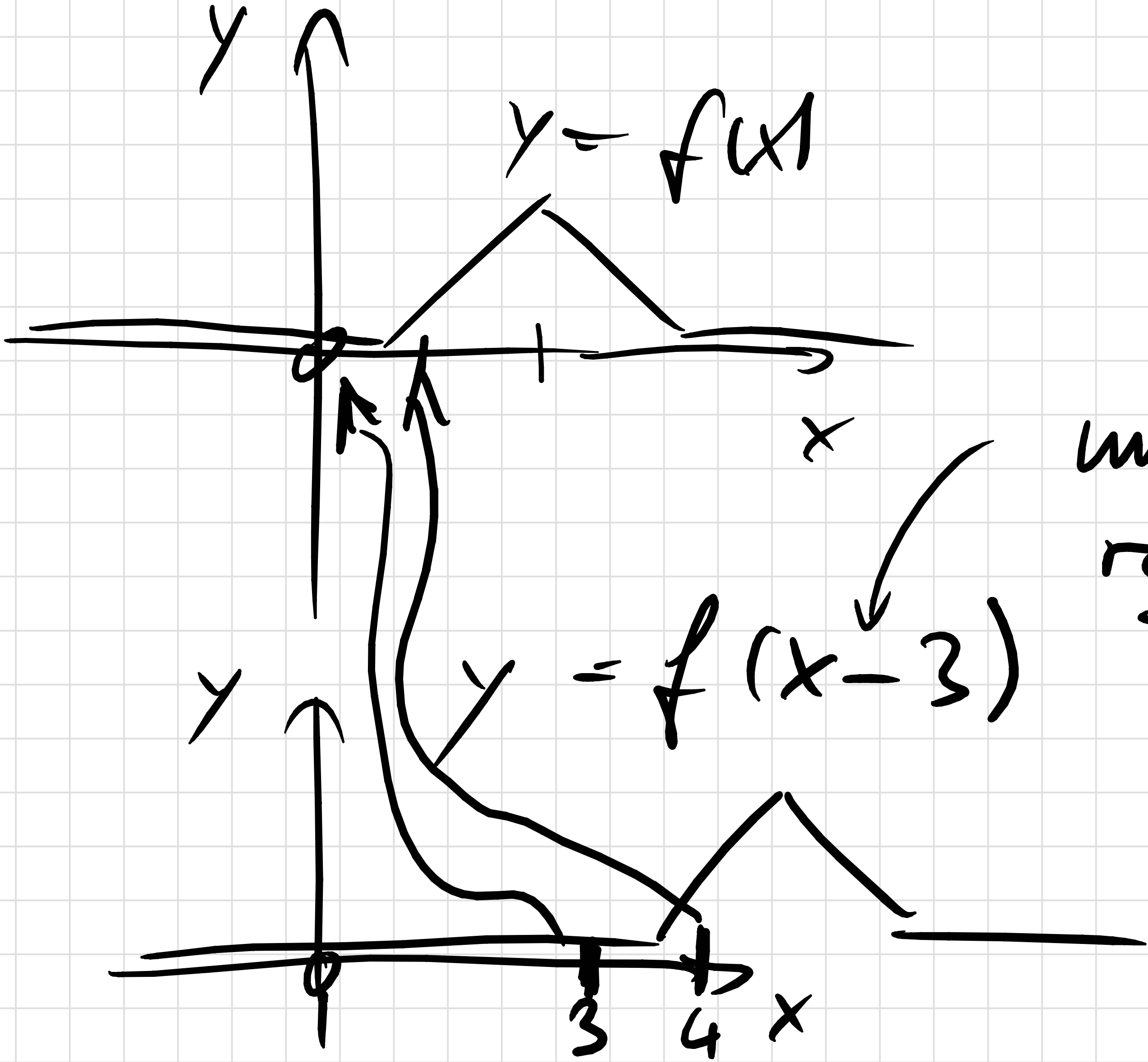
$y=f(x)+3$



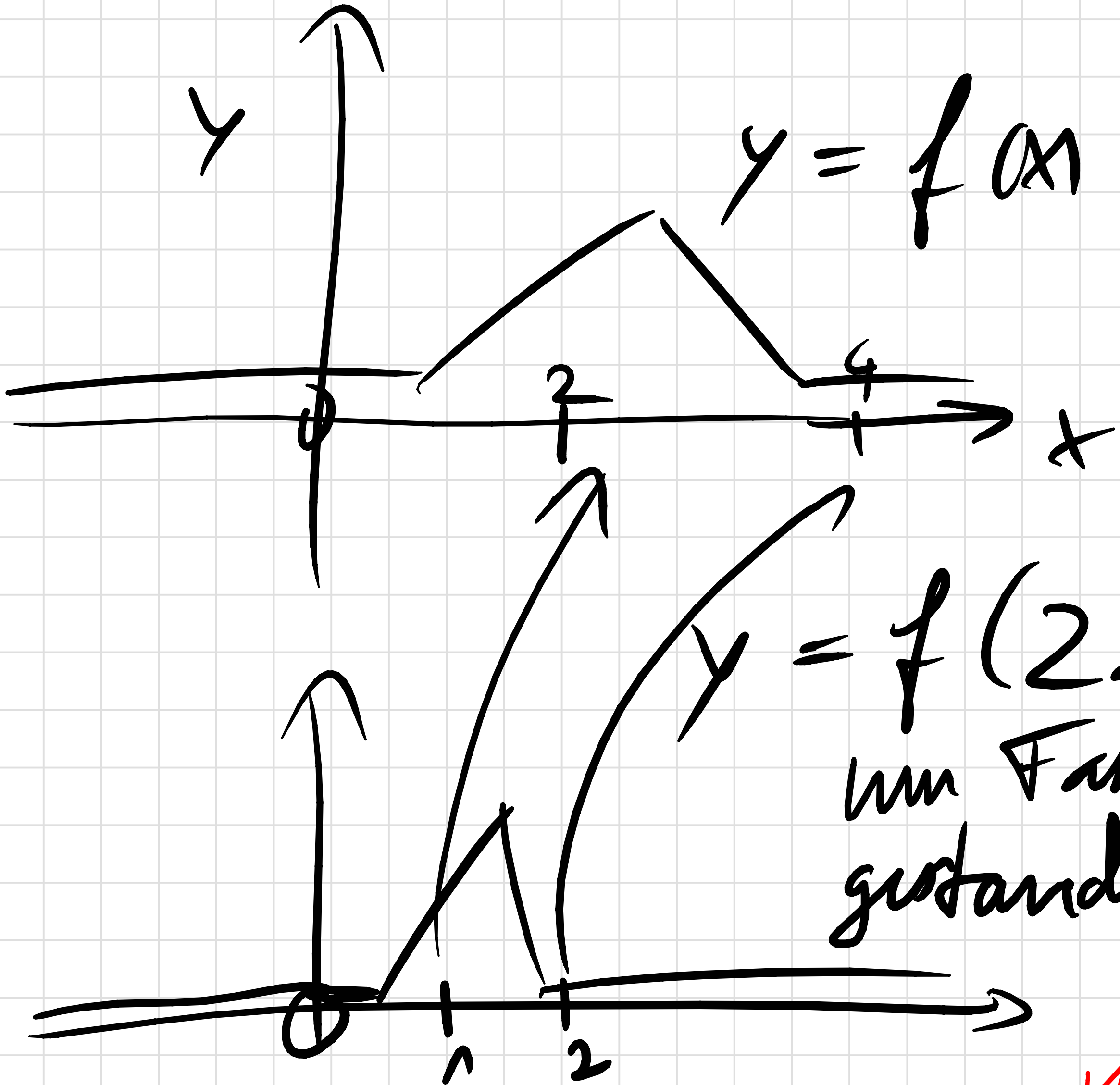
$y=2f(x)$

$y=2f(x)+3$

<u>erst</u> strecken,
<u>dann</u> schieben



um 3 nach **rechts** verschieben



um Faktor 2 gestaucht

$$y = f(2x+3)$$

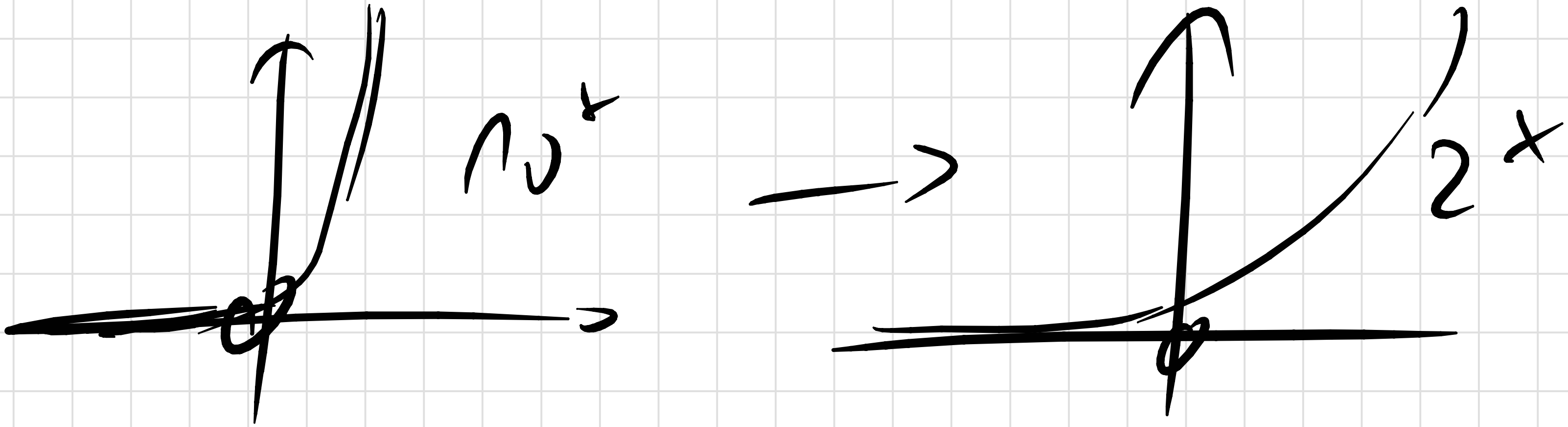
erst ~~stauchen~~ nach links,
dann ~~nach links~~ stauchen

$y = f(13(x-4))$ erst ~~nach rechts~~ stauchen, dann ~~stauchen~~ nach rechts

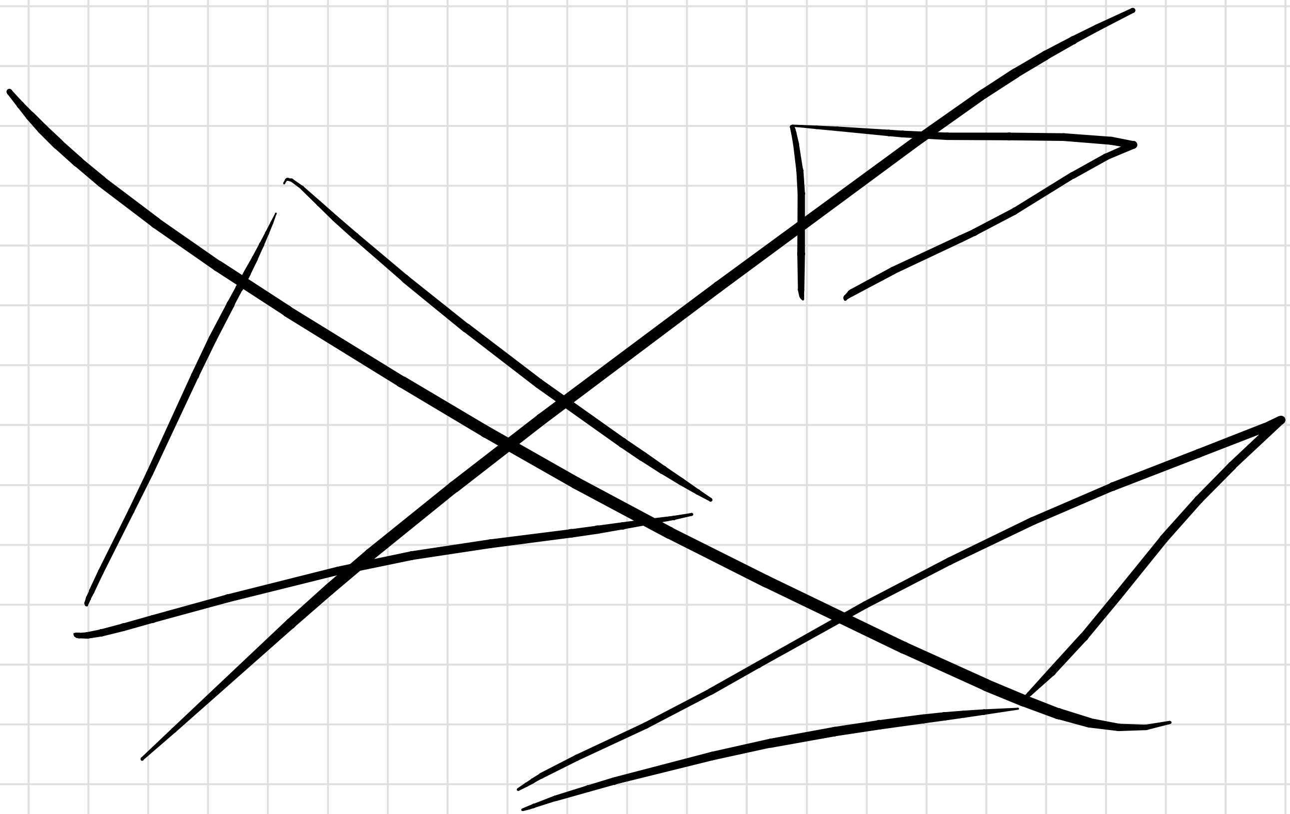
---

z.B.: $2^x = 10^{\log_{10} 2^x} = 10^{x \underbrace{\log_{10} 2}_{\approx \frac{1}{3}}}$

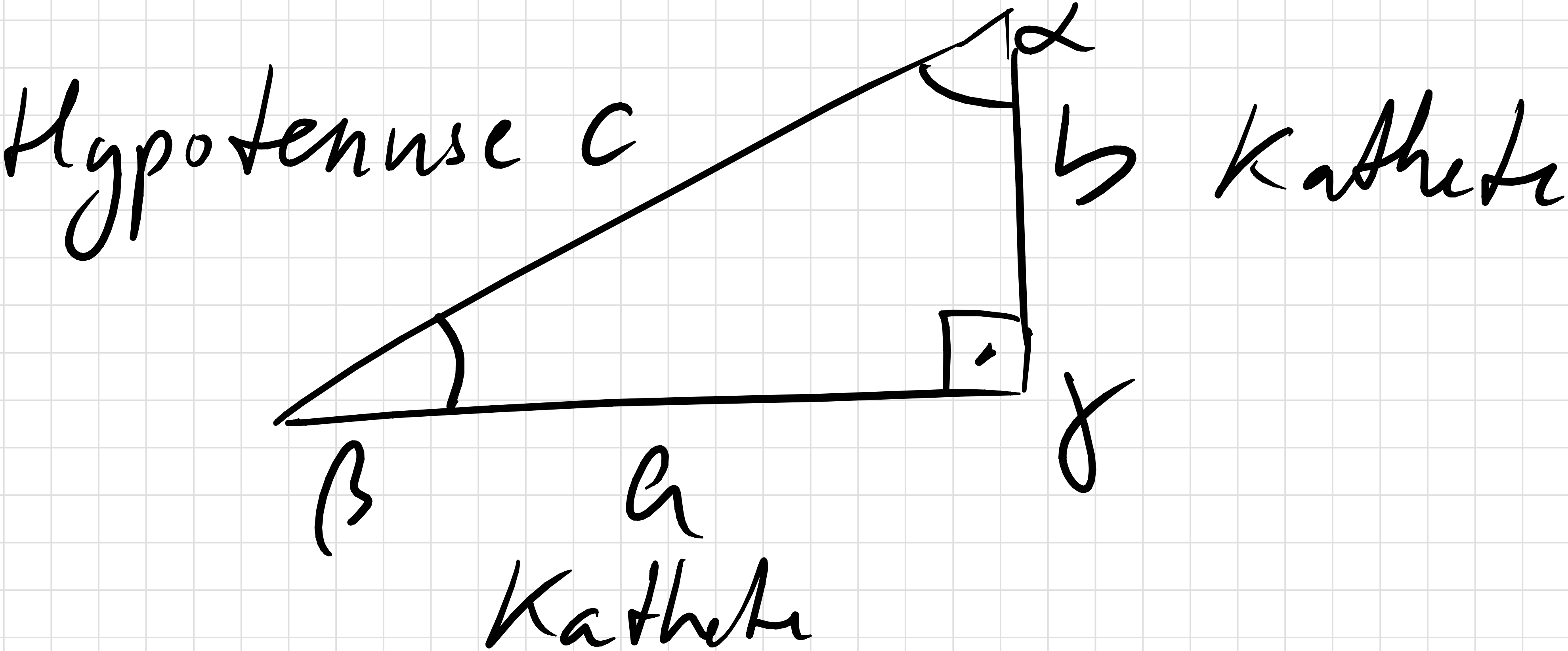
Nehme Kurve $y = 10^x$, strecke um Faktor ≈ 3, das gibt die Kurve $y = 2^x$.

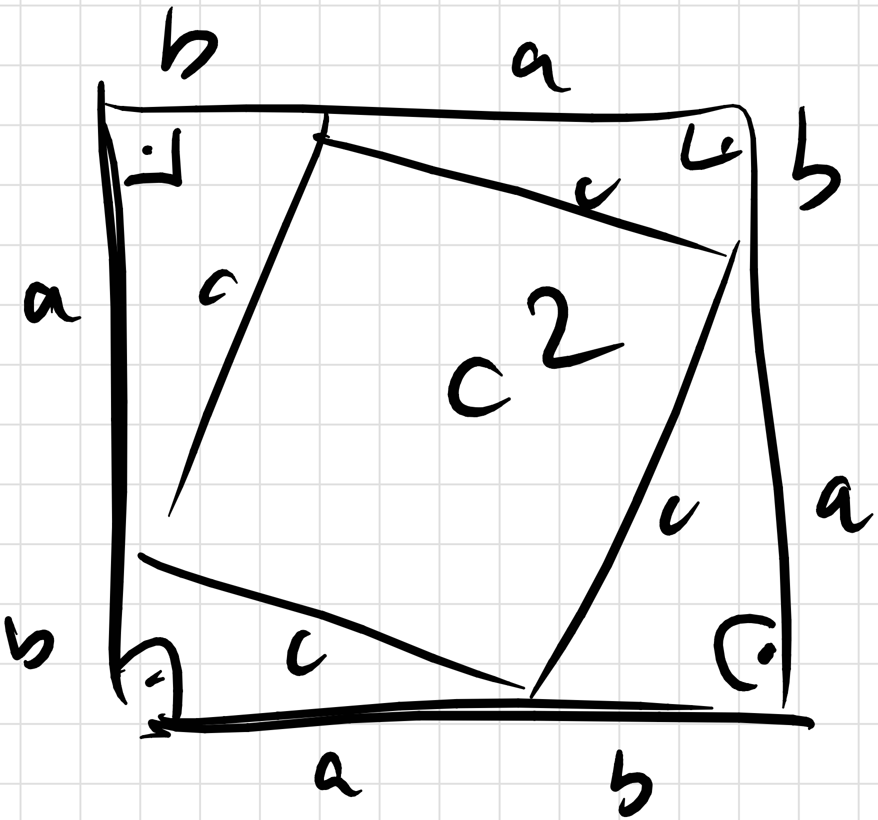

10^x
2^x

Trigonometrie : Sinus & Freunde



rechtwinkliges Dreieck:



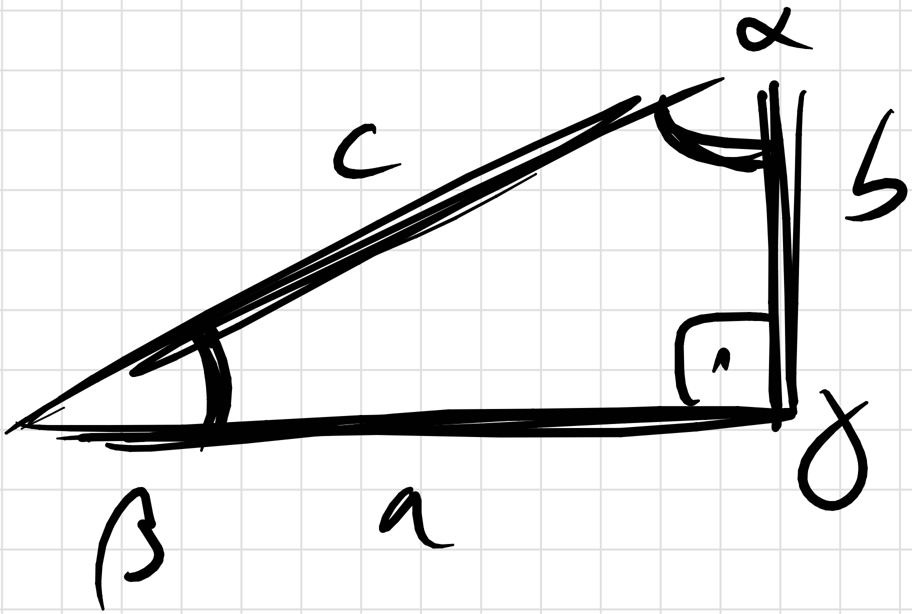


$$(a+b)^2 = a^2 + \cancel{2ab} + b^2$$

$$= 4 \cdot \frac{1}{2} ab$$

$$+ c^2$$

$$= \cancel{2ab} + c^2$$



$$\sin(\alpha) = \frac{\text{Gegenkathete}}{\text{Hypotenuse}} = \frac{a}{c}$$

$$\sin(\beta) = \frac{b}{c}$$

$$=$$

$$\cos(\alpha) = \frac{\text{Ankathete}}{\text{Hypotenuse}} = \frac{b}{c}$$

$$\cos(\beta) = \frac{a}{c}$$

$$\tan(\alpha) = \frac{\text{Gegenkathete}}{\text{Ankathete}} = \frac{a}{b}$$

$$= \frac{a/c}{b/c} = \frac{\sin(\alpha)}{\cos(\alpha)}$$

$$\cot(\alpha) = \frac{b}{a} = \frac{\cos(\alpha)}{\sin(\alpha)}$$

Pythagoras:

$$a^2 + b^2 = c^2 \quad // :c^2$$

$$\Rightarrow \quad \frac{a^2}{c^2} + \frac{b^2}{c^2} = 1$$

$$\Rightarrow \quad \left(\frac{a}{c}\right)^2 + \left(\frac{b}{c}\right)^2 = 1$$

$\sin(\alpha)$ $\cos(\alpha)$

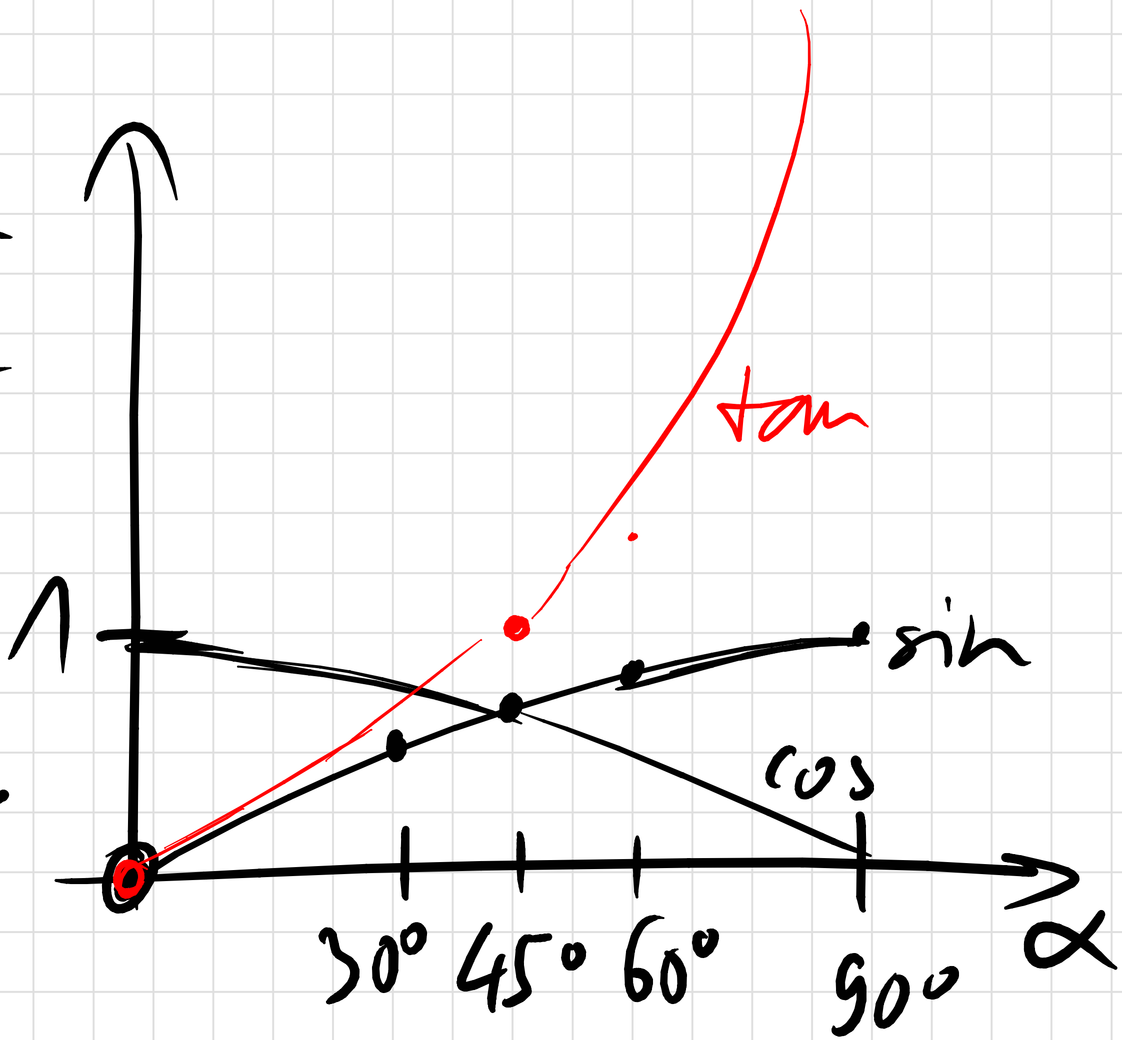
$$\Rightarrow \quad \sin(\alpha)^2 + \cos(\alpha)^2 = 1$$

$$\frac{1}{1+\tan^2} = \frac{1}{1+\left(\frac{\sin}{\cos}\right)^2} = \frac{1}{1+\frac{\sin^2}{\cos^2}}$$

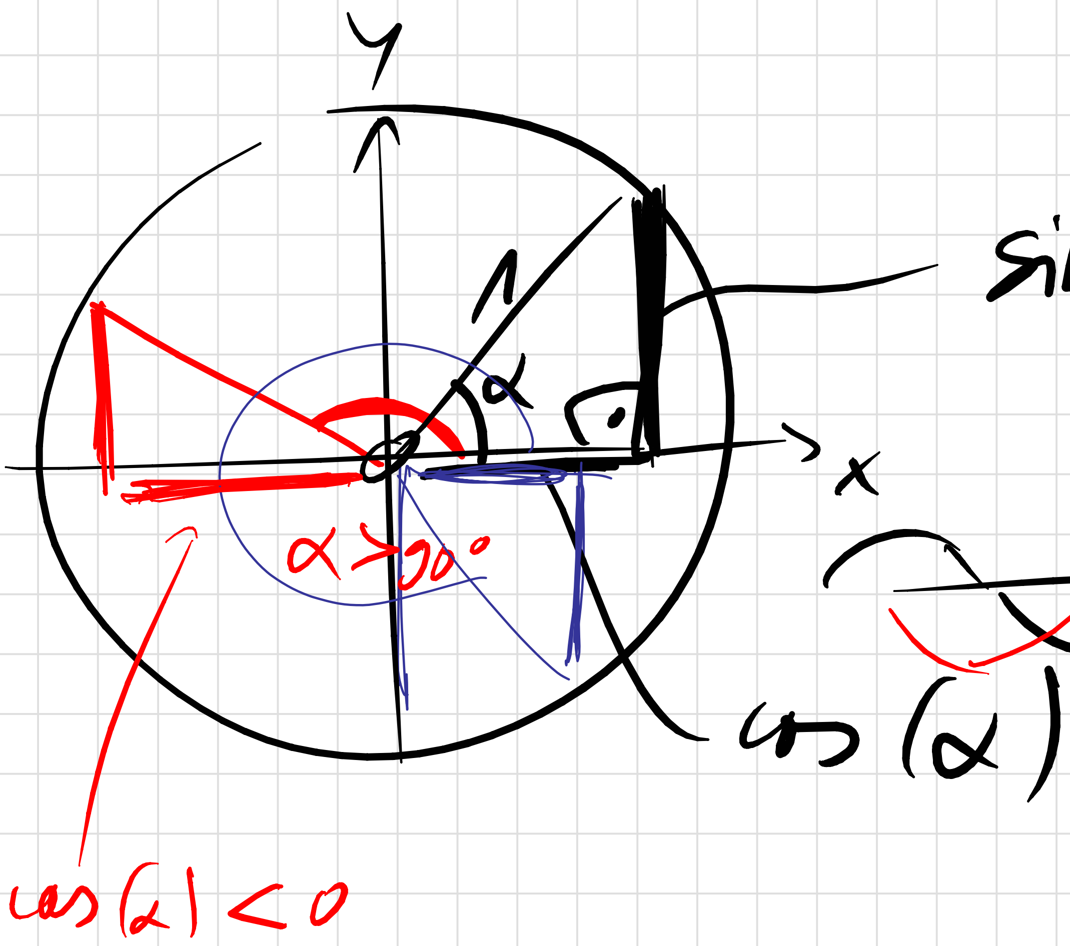
$$= \frac{\cos^2}{\underbrace{\cos^2+\sin^2}_{1}} = \cos^2$$

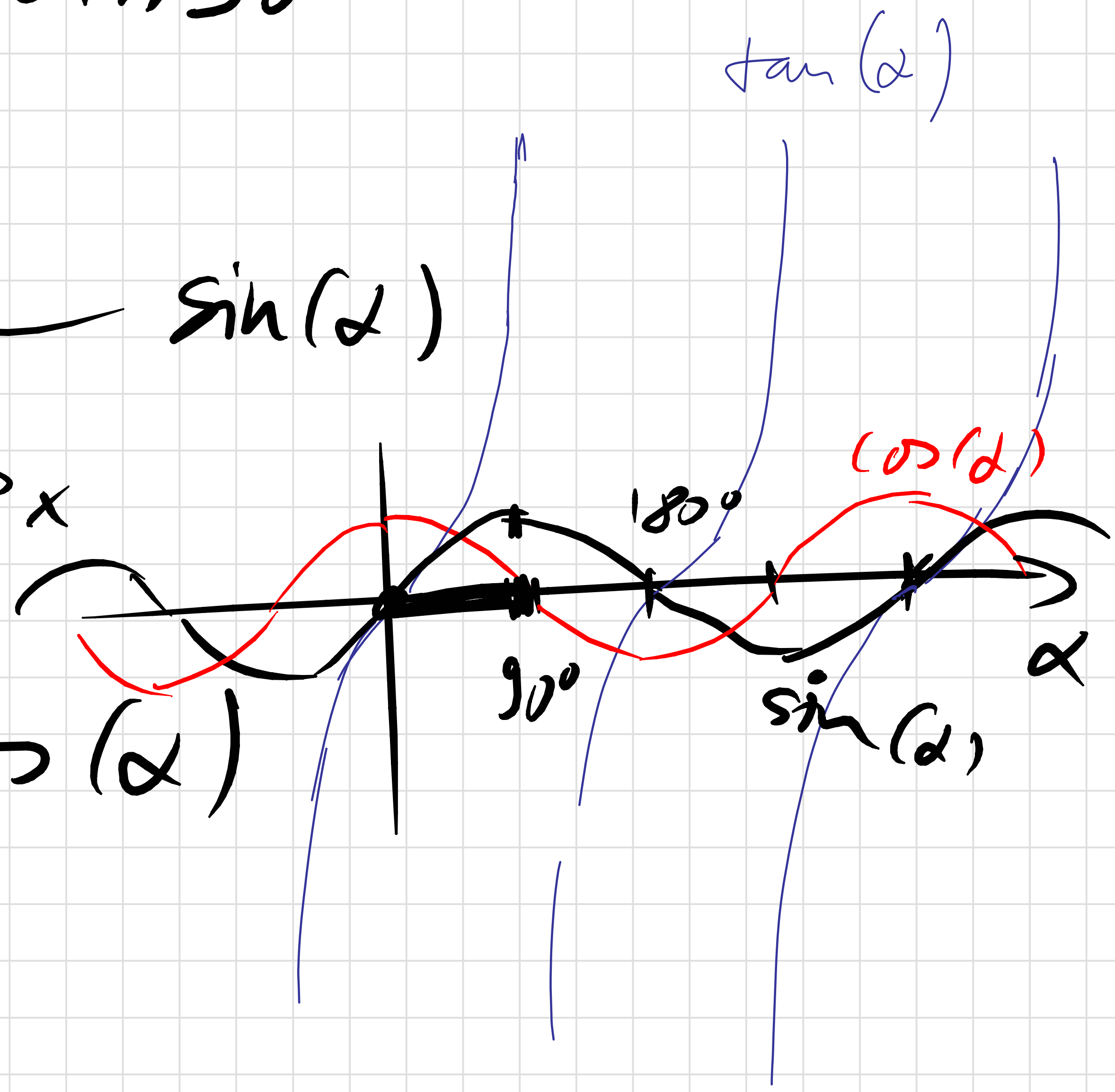
Werte von sin, cos, tan

	sin	cos	tan
0°	0	1	0
30°	$1/2$	$\sqrt{3}/2$	$1/\sqrt{3}$
45°	$1/\sqrt{2}$	$1/\sqrt{2}$	1
60°	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$
90°	1	0	n.d.

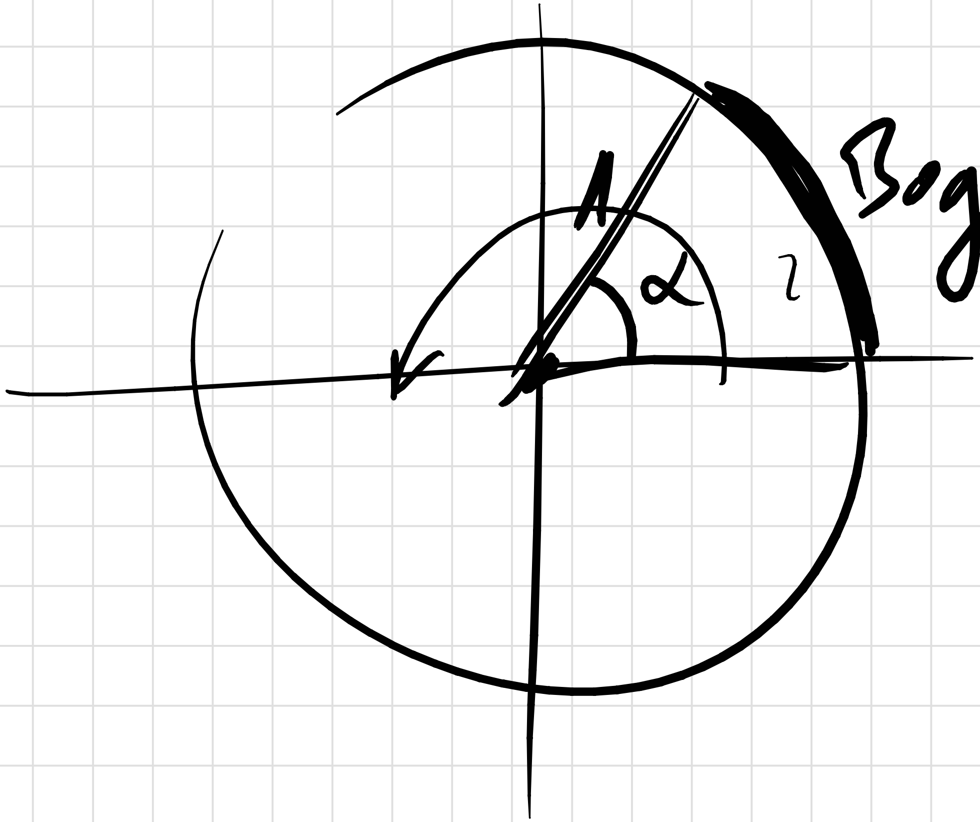


Jenseits von 0°...90°



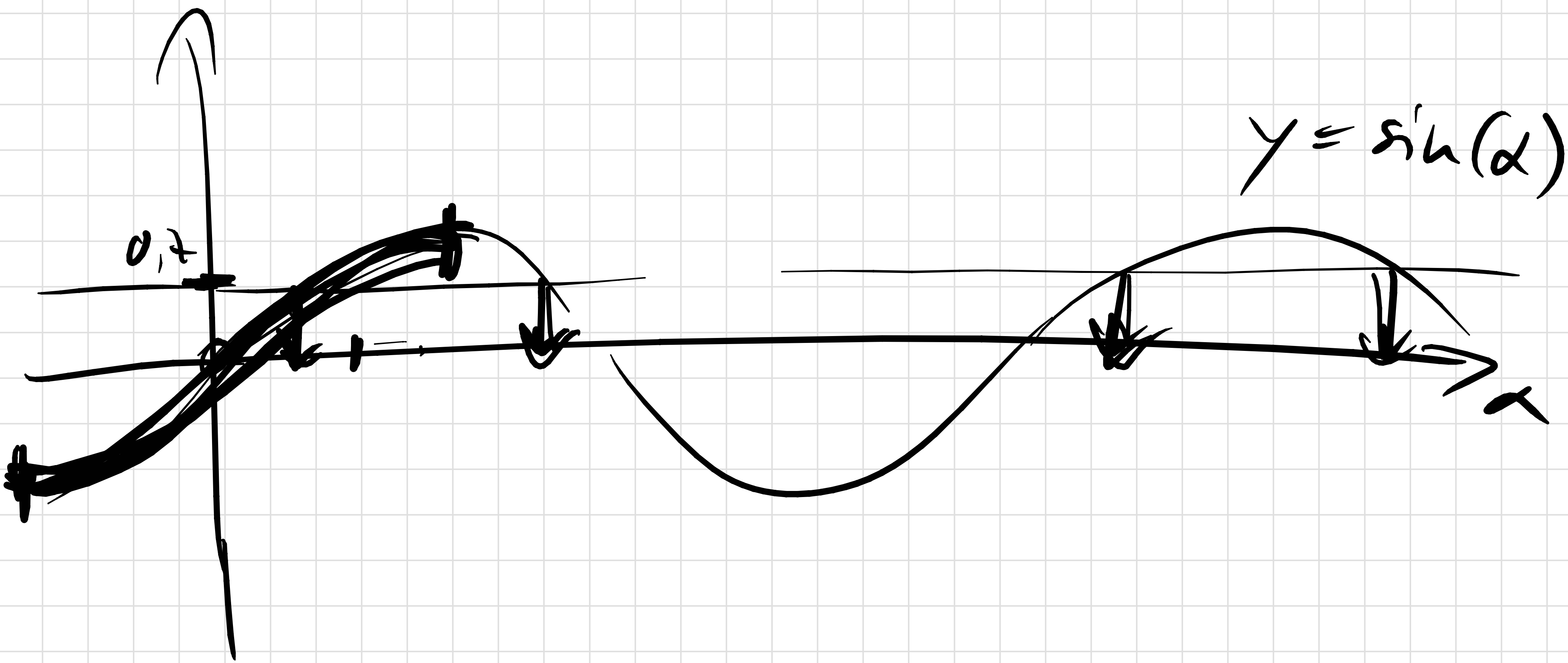


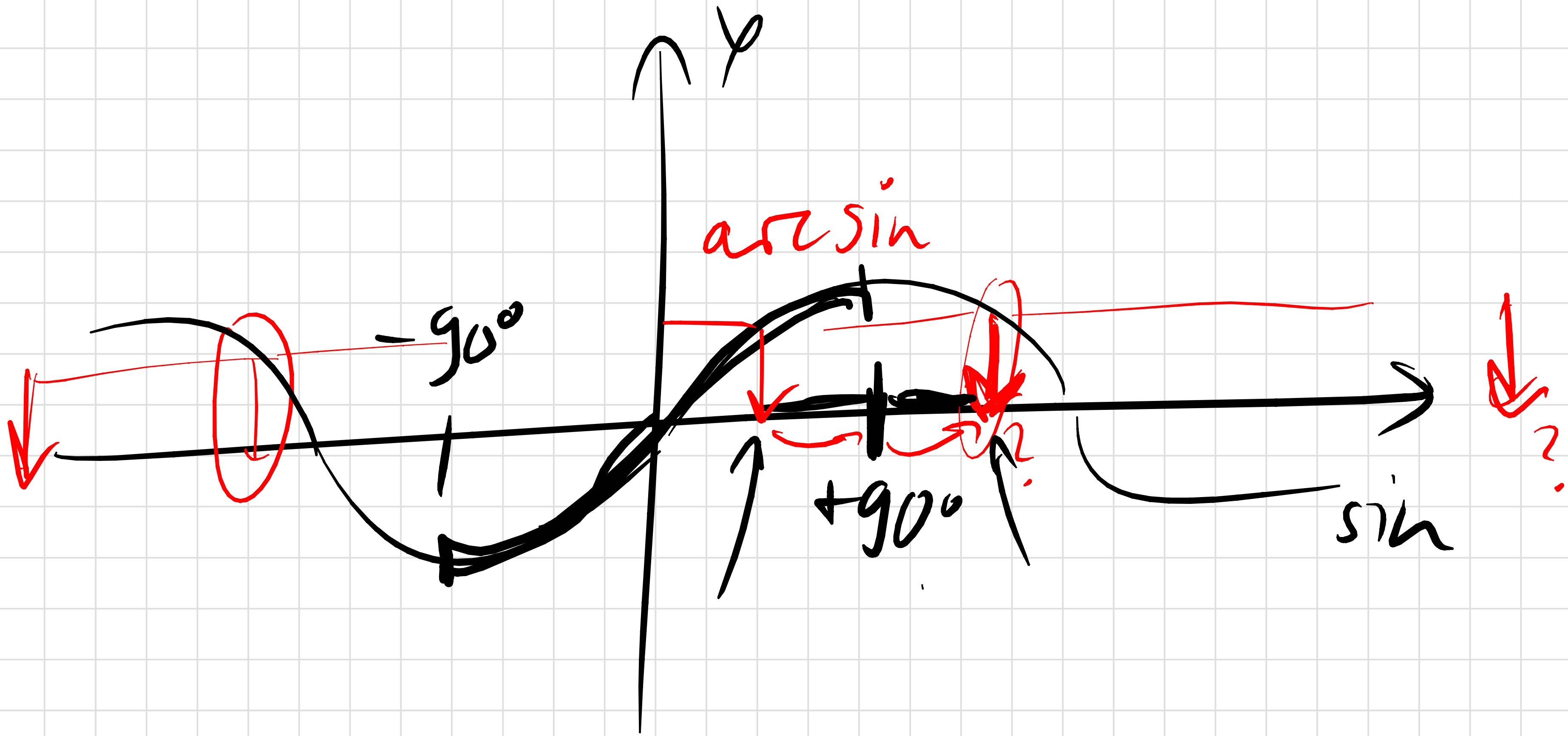
Gradmaß ⟶ Bogenmaß



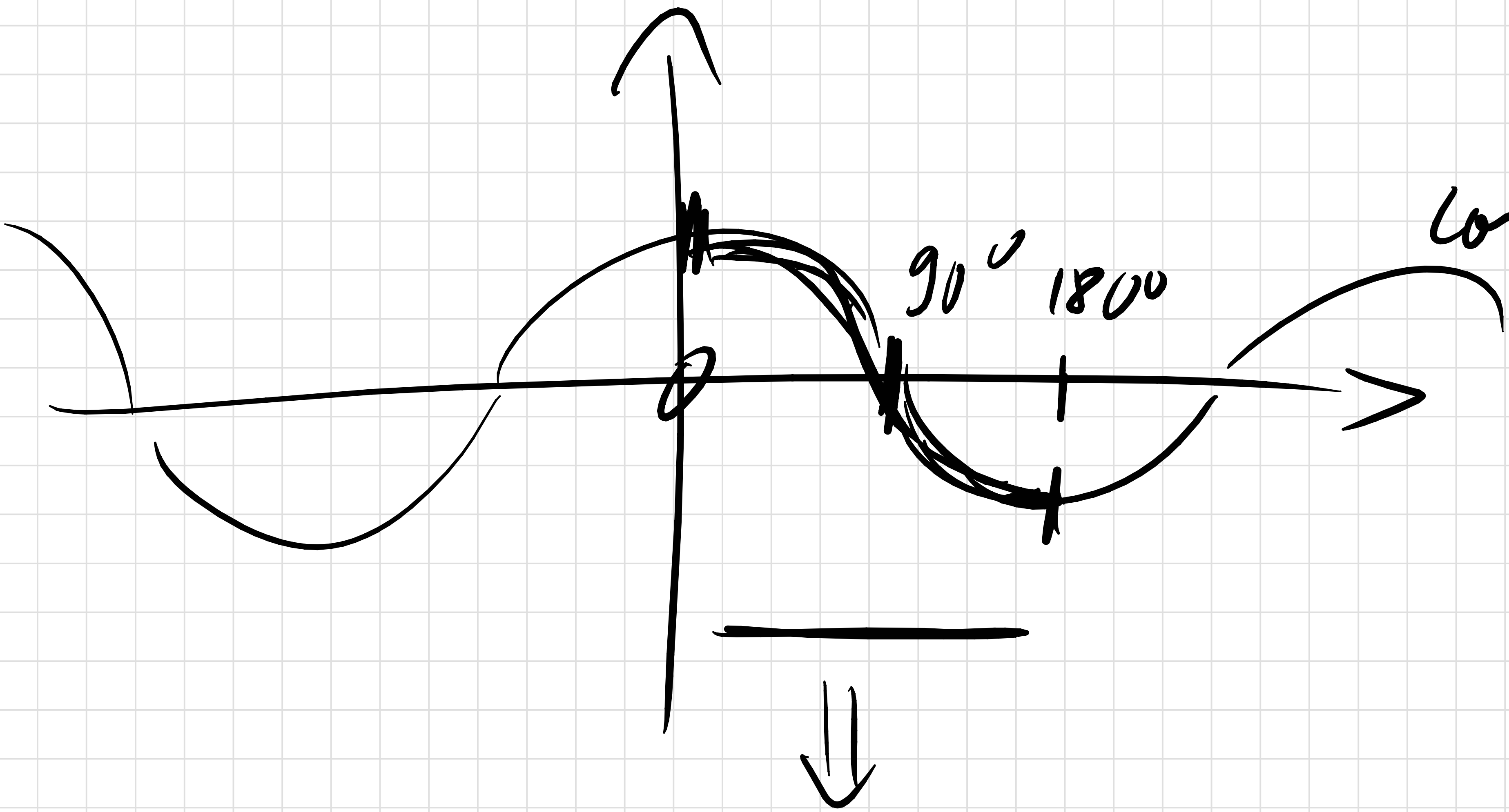
Grad	Bogenmaß
60°	$\pi/3 \approx 1$
90°	$\pi/2 \approx 1{,}6$
180°	$\pi = 3{,}14\ldots$
360°	$2\pi = 6{,}28\ldots$

Arcusfunktionen

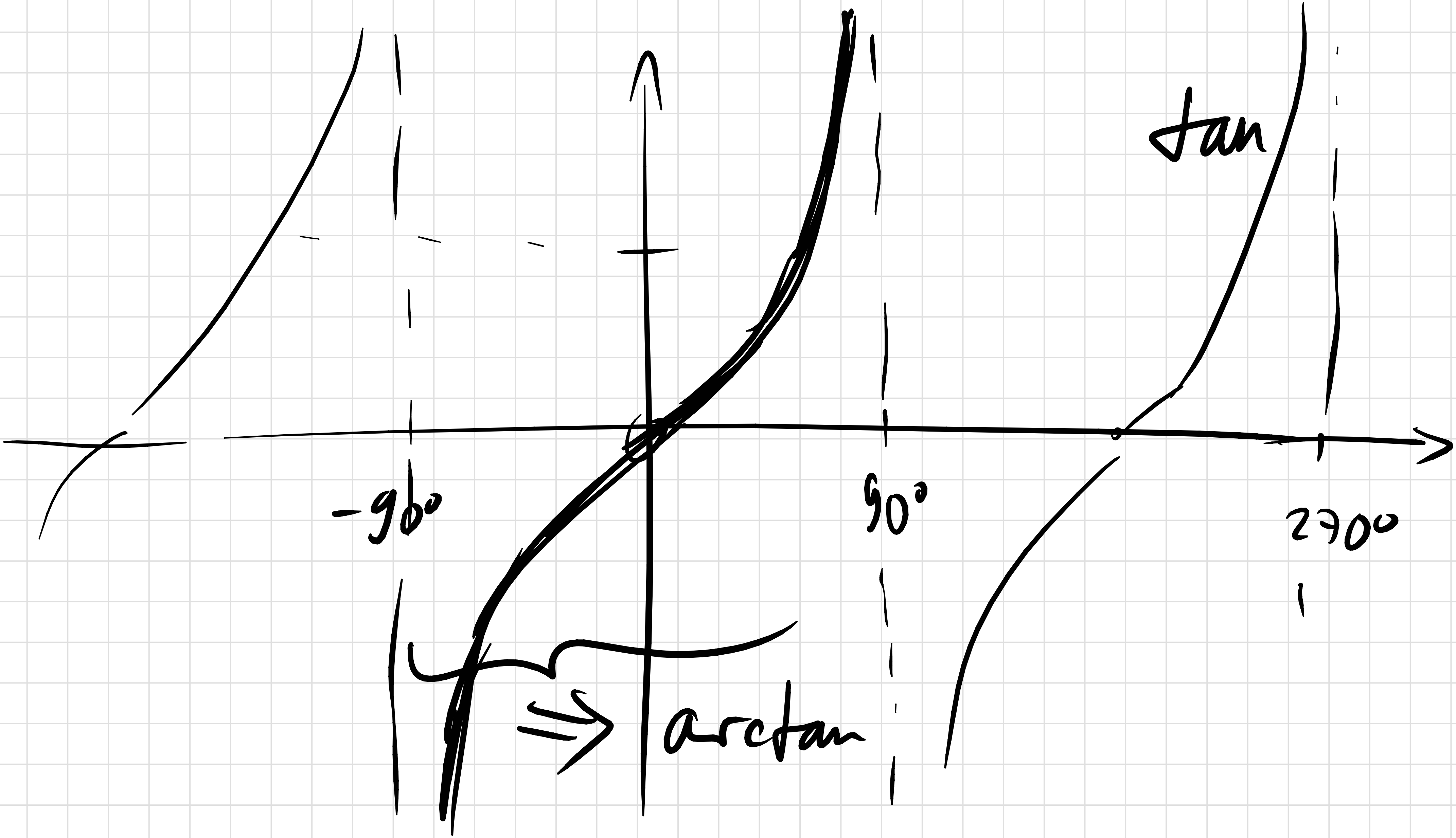




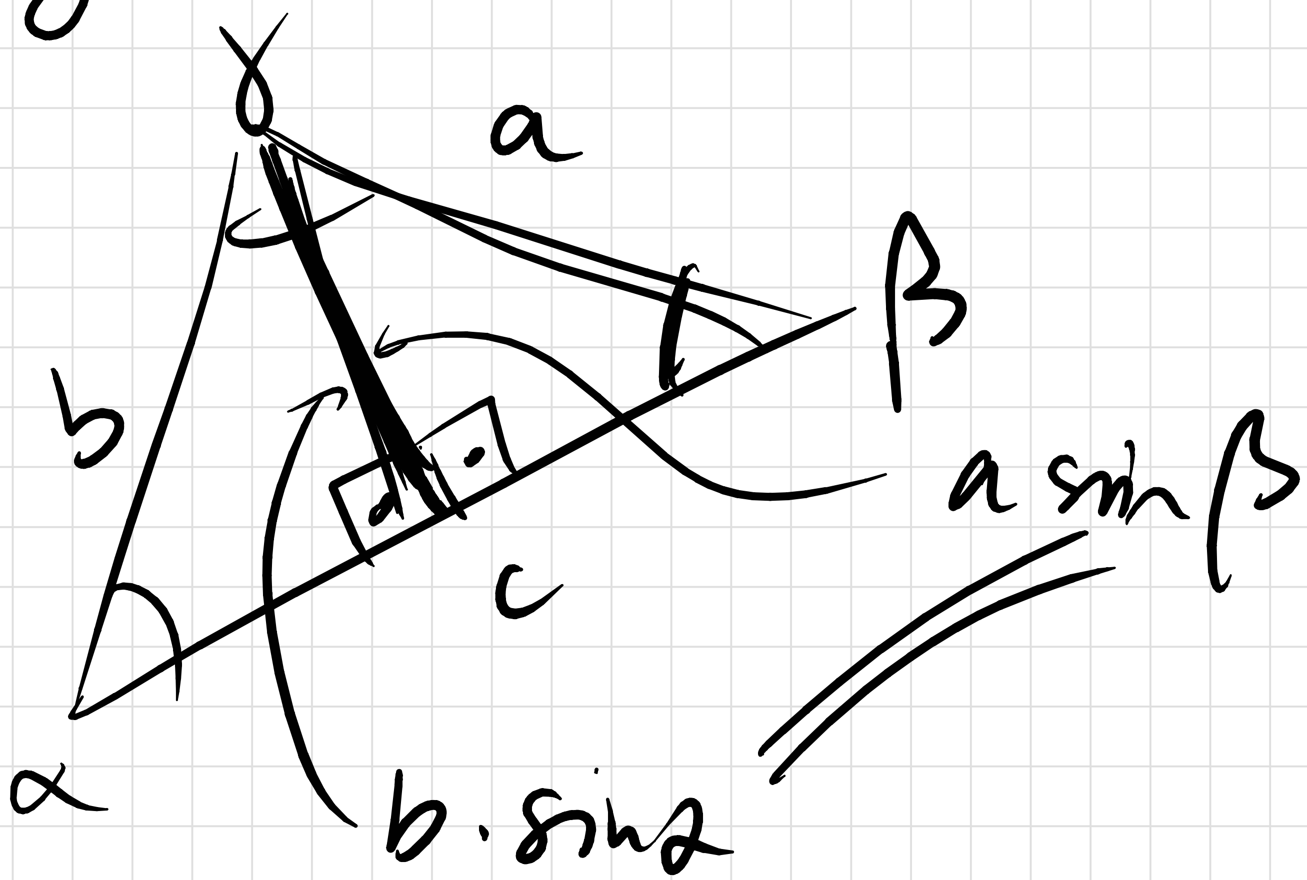
y
arcsin
-90°
+90°
?
?
sin



arccos


tan
-90°
90°
270°
arctan

Allgemeine Dreiecke

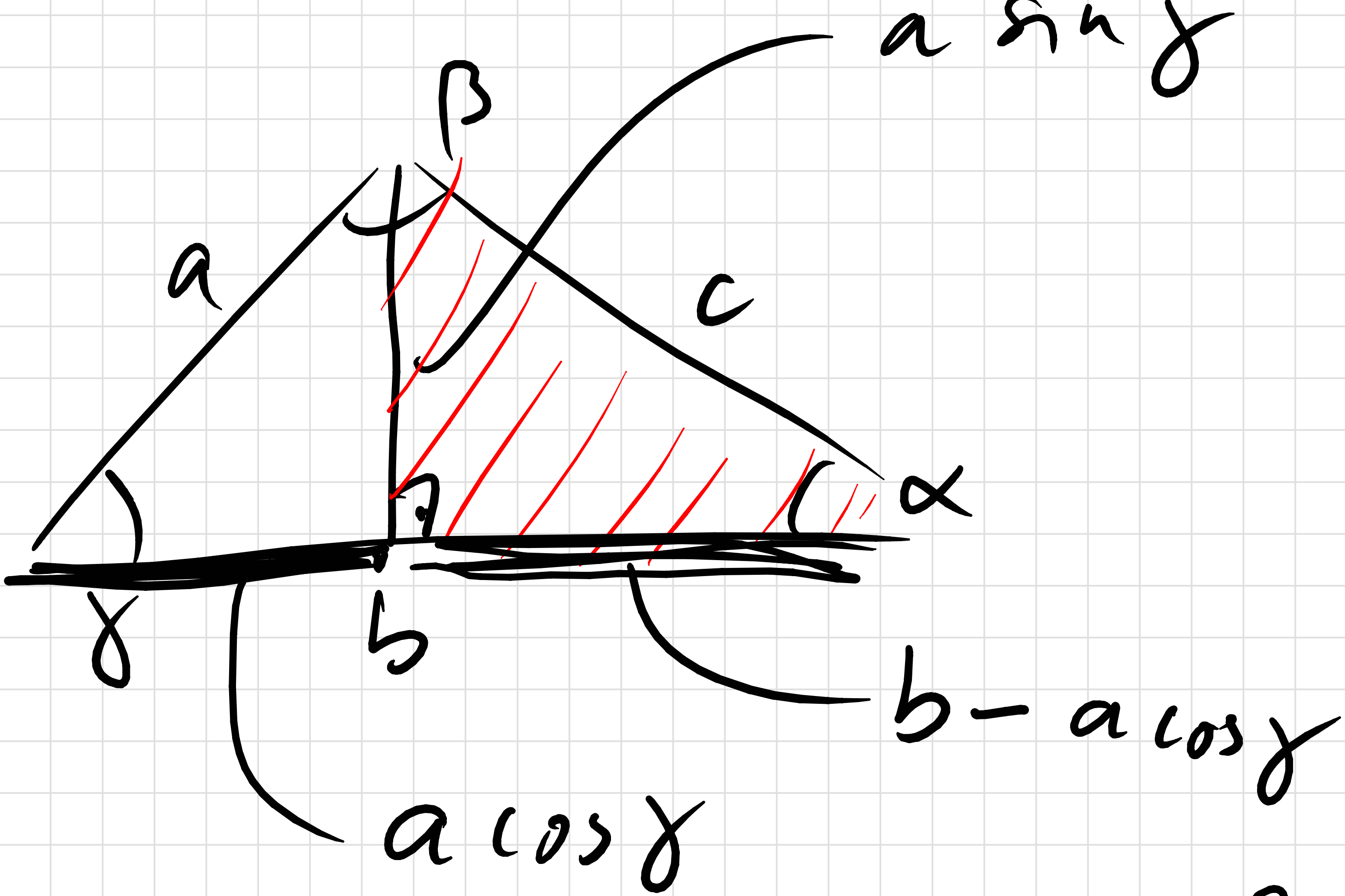


$$b \sin\alpha = a \sin\beta \implies \frac{\sin\alpha}{a} = \frac{\sin\beta}{b}$$

$$\Rightarrow \frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$$

Sinussatz

Cosinussatz



$$c^2 = (a \sin \gamma)^2 + (b - a\cos\gamma)^2$$

$$= a^2 (\sin\gamma)^2 + b^2 - 2ab\cos\gamma$$
$$+ a^2 (\cos\gamma)^2$$

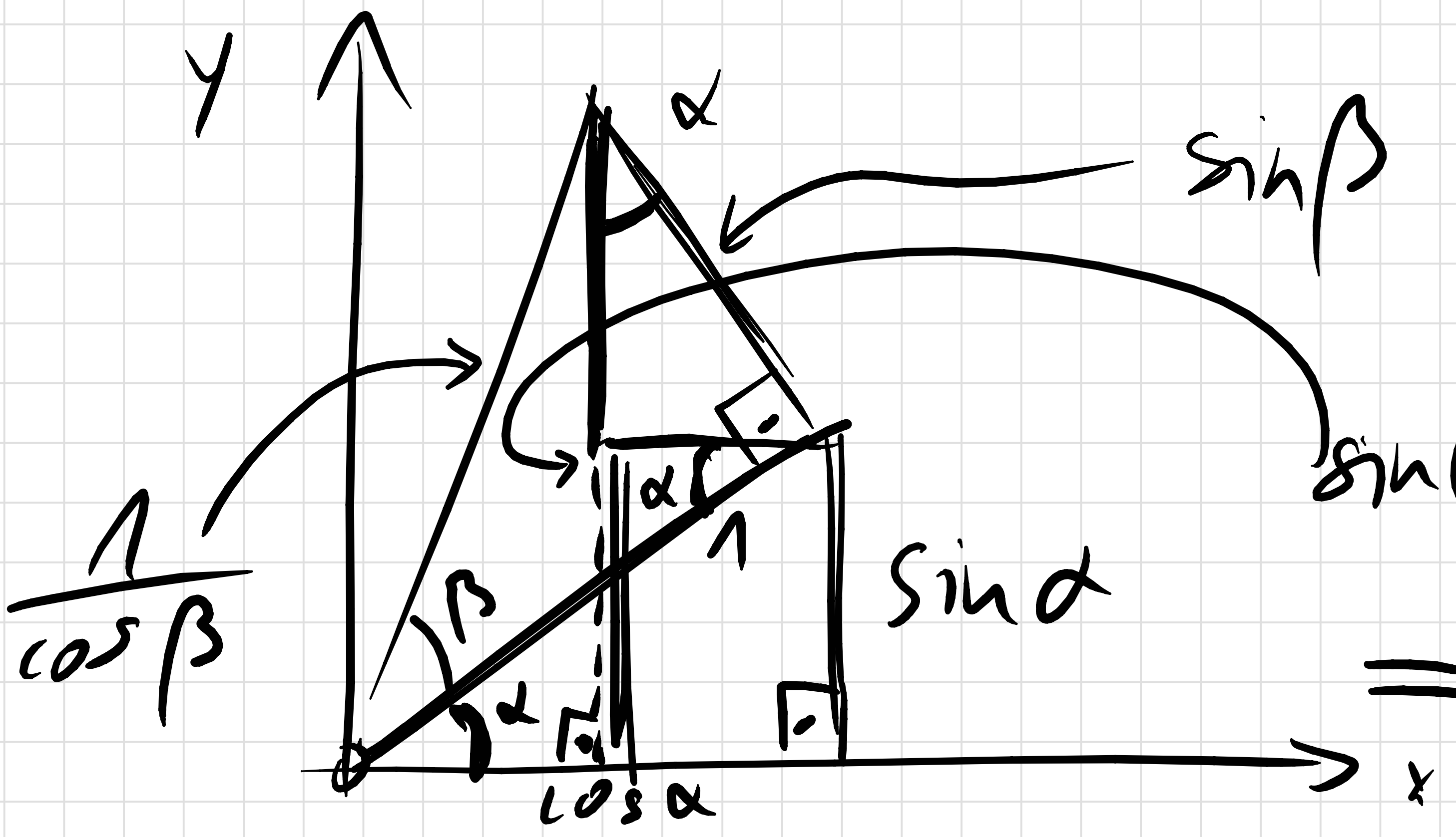
$$= a^2 \cdot 1 + b^2 - 2ab\cos\gamma$$

$$= a^2 + b^2 - 2ab\cos\gamma.$$

Additionstheoreme

$$\sin(\alpha+\beta) = ?$$

$$\cos(\alpha+\beta) = ?$$



$$\sin\beta \cdot \frac{1}{\cos\beta}$$

$$\sin(\alpha+\beta) \cdot \frac{1}{\cos\beta} = \sin\alpha$$

$$+\frac{\cos\alpha \cdot \sin\beta}{\cos\beta}$$

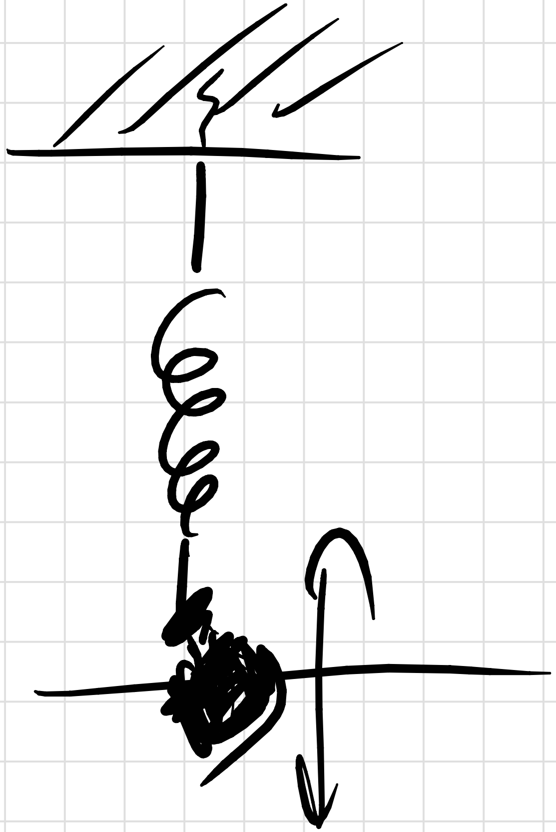
$$\frac{\sin(\alpha+\beta)}{\cos\beta} = \sin\alpha + \frac{\cos\alpha \cdot \sin\beta}{\cos\beta}$$

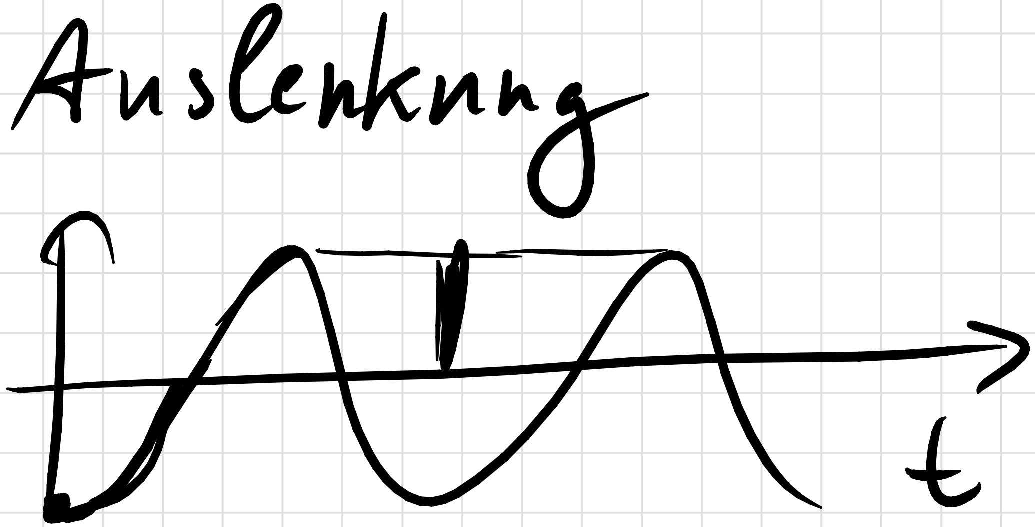
$$\Rightarrow \sin(\alpha+\beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$$

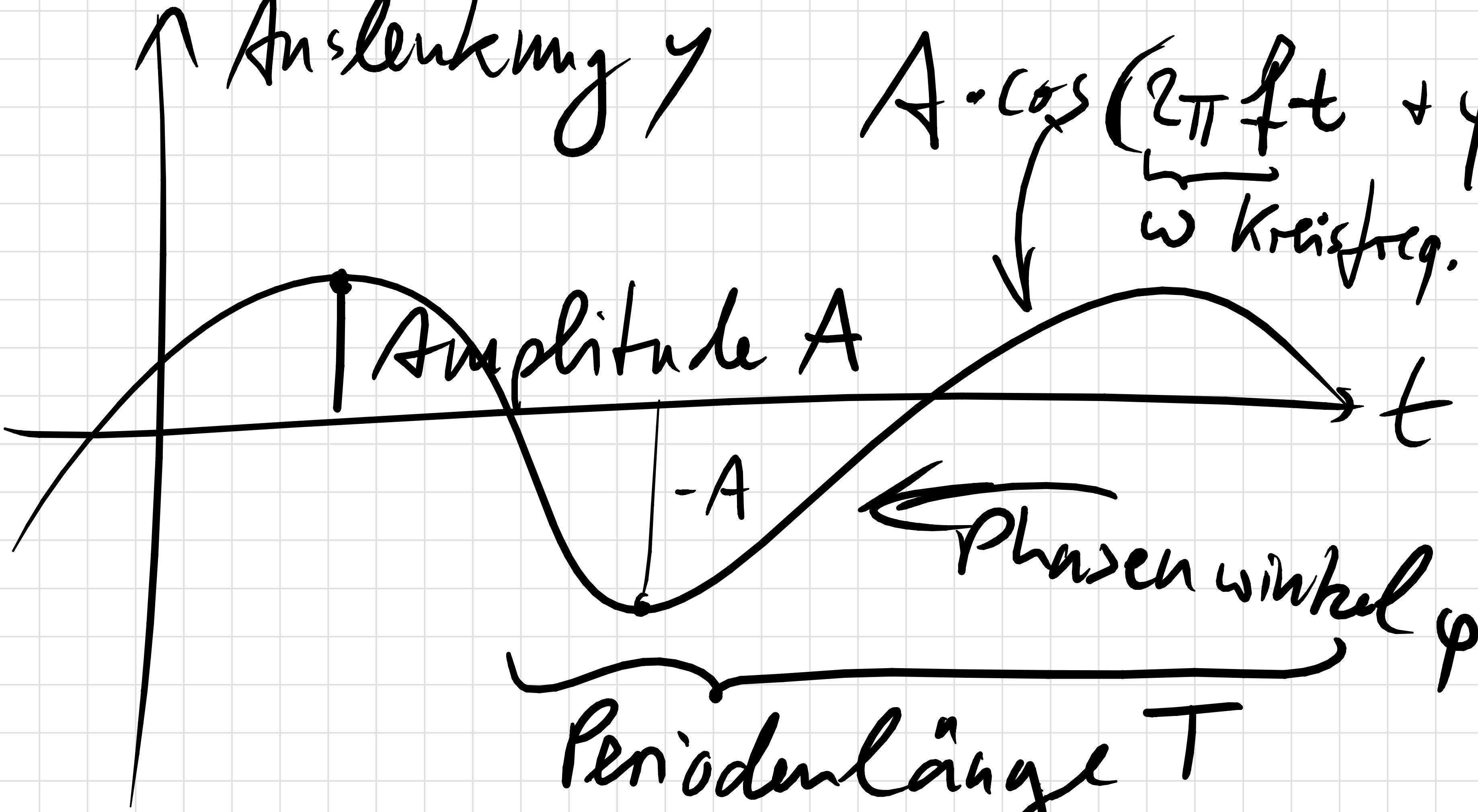
ebenso:

$$\cos(\alpha+\beta) = \cos(\alpha)\cdot\cos(\beta) - \sin(\alpha)\cdot\sin(\beta)$$

Schwingungen







$$A \cdot \cos(\underbrace{2\pi f}_{\omega \text{ Kreisfreq.}} t + \varphi)$$

Frequenz $f = \frac{1}{T}$