Potenzen

Basis Exponent

$$3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 3^5$$

$$3^2 = 3 \cdot 3$$

$$3^{5} \cdot 3^{4}$$

$$3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$$

$$= 3^{9} = 3^{5+4}$$

$$\boxed{a^{n} \cdot a^{m} = a^{n+m}}$$

zunächst nur

für $n, m = 1, 2, 3, 4, \ldots$

$$3^4 \cdot 3^5 = 3^9$$

$$3^0 = ?$$

Wollen:

$$\boxed{3^0} \cdot \underline{3^4} \overset{!}{=} 3^{0+4} = \underline{3^4}$$

muss 1 sein!

$$\boxed{0^0 = 1}$$

$$0{,}00000\ldots\infty 1^0 = 1$$

$$3^{-4}$$

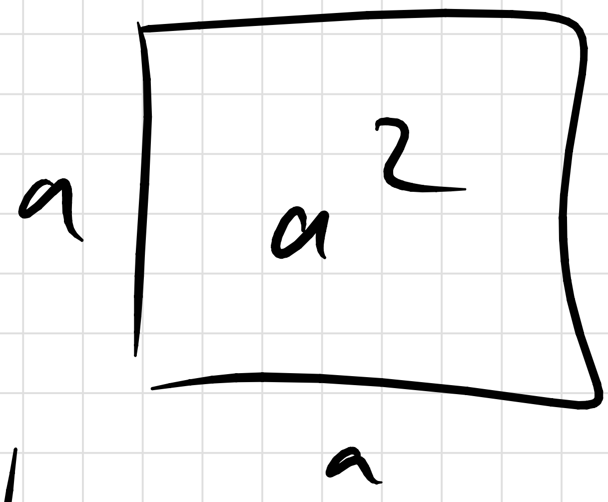
$$3^{-4} \cdot 3^{4} \overset{!}{=} 3^{\overbrace{-4+4}^{0}} = 3^{0}$$
$$= 1$$

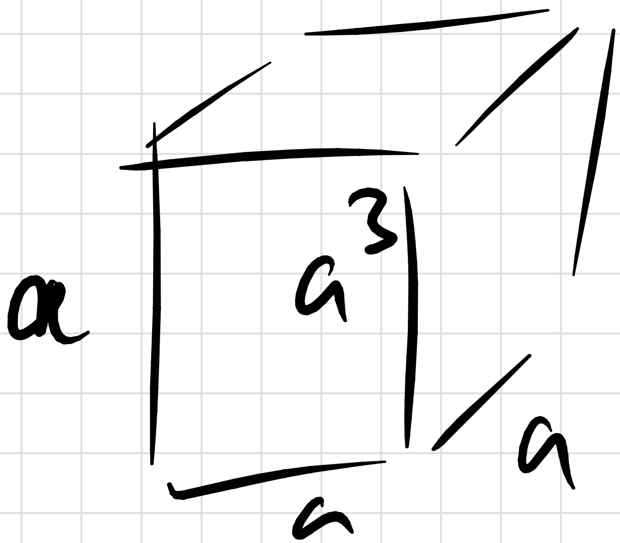
$$\Rightarrow 3^{-4} = \frac{1}{3^{4}}$$

$$a^{-n} := \frac{1}{a^n}$$

für $a \neq 0$

Beispiele:





Passwort 8 Zeichen aus 52

52 52 52

a ... z

A ... Z

Möglichkeiten = 52^8

a a a a a a a a

a a a a a a a b

... Z Z Z Z Z Z Z Z

$$(3 \cdot 5)^4 = 3 \cdot 5 \cdot 3 \cdot 5 \cdot 3 \cdot 5 \cdot 3 \cdot 5$$

$$= 3 \cdot 3 \cdot 3 \cdot 3 \cdot 5 \cdot 5 \cdot 5 \cdot 5$$

$$= 3^4 \cdot 5^4$$

$$(ab)^n = a^n \cdot b^n$$

$$(-1)^0 = 1$$

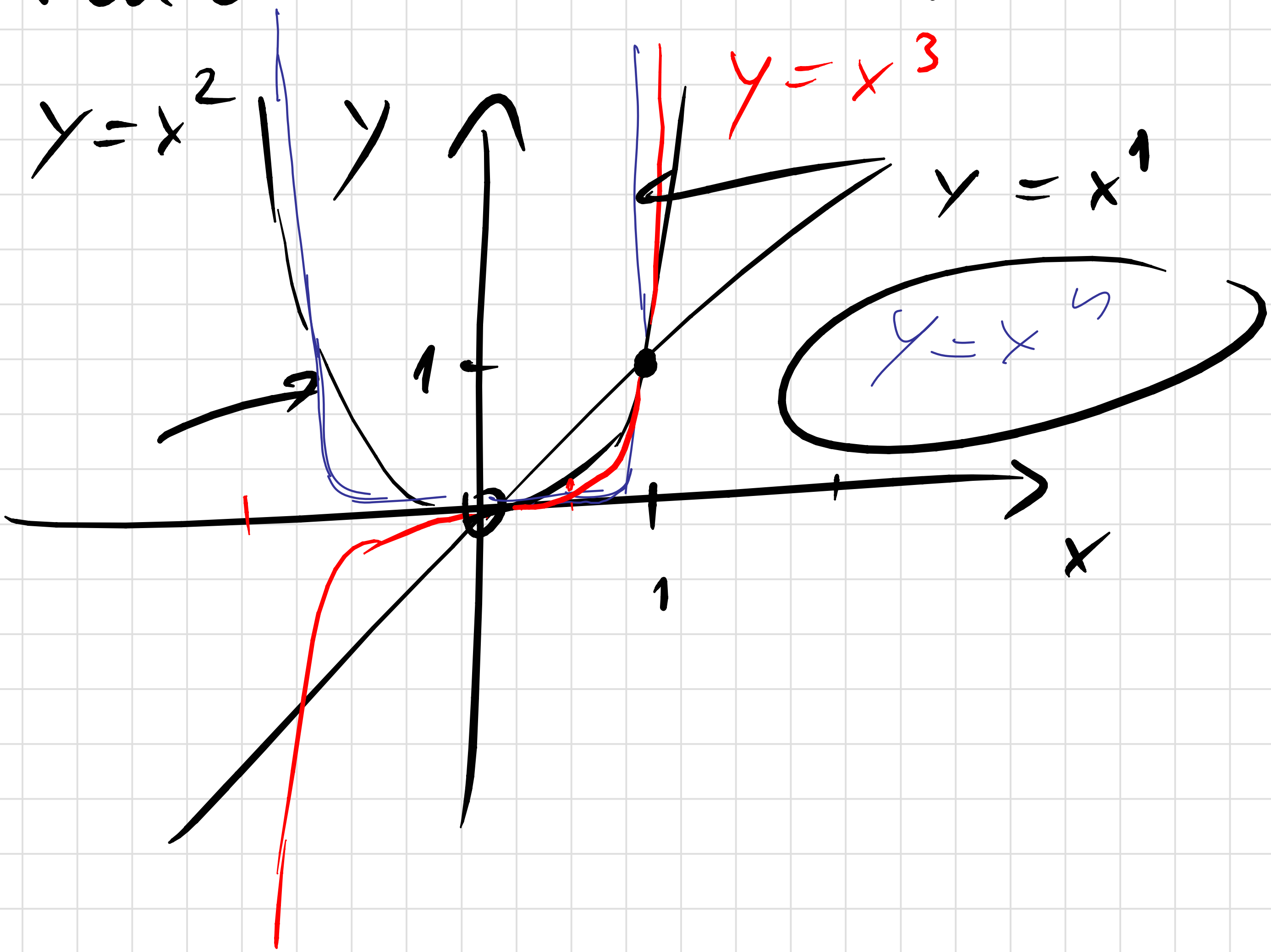
$$(-1)^1 = -1$$

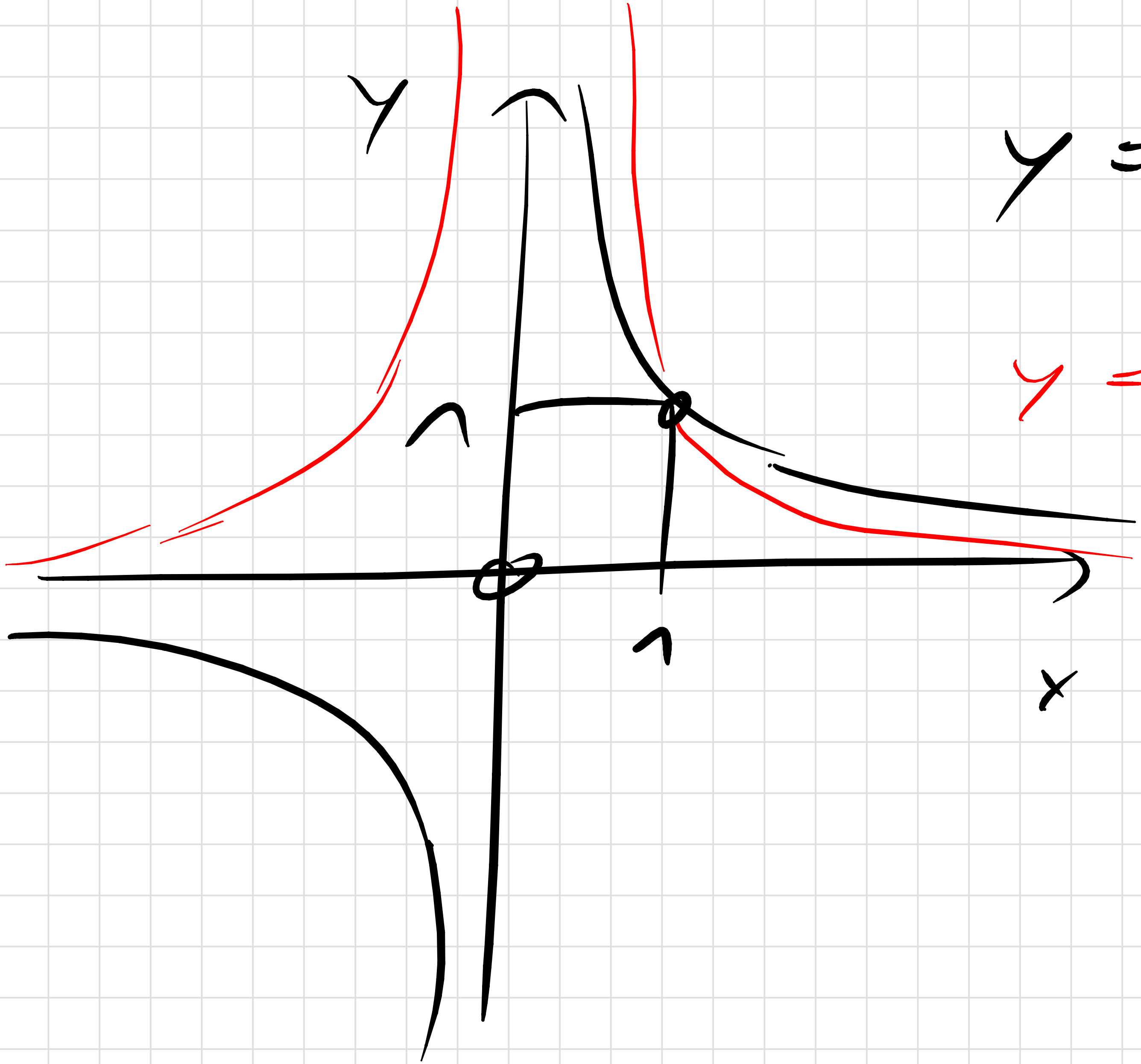
$$(-1)^2 = 1$$

$$(-1)^3 = -1$$

$$(-1)^n = \begin{cases} 1, & n \text{ gerade} \\ -1, & n \text{ ungerade} \end{cases}$$

Potenzen als Kurven





$$y = x^{-1} = \frac{1}{x}$$

$$y = x^{-2} = \frac{1}{x^2}$$

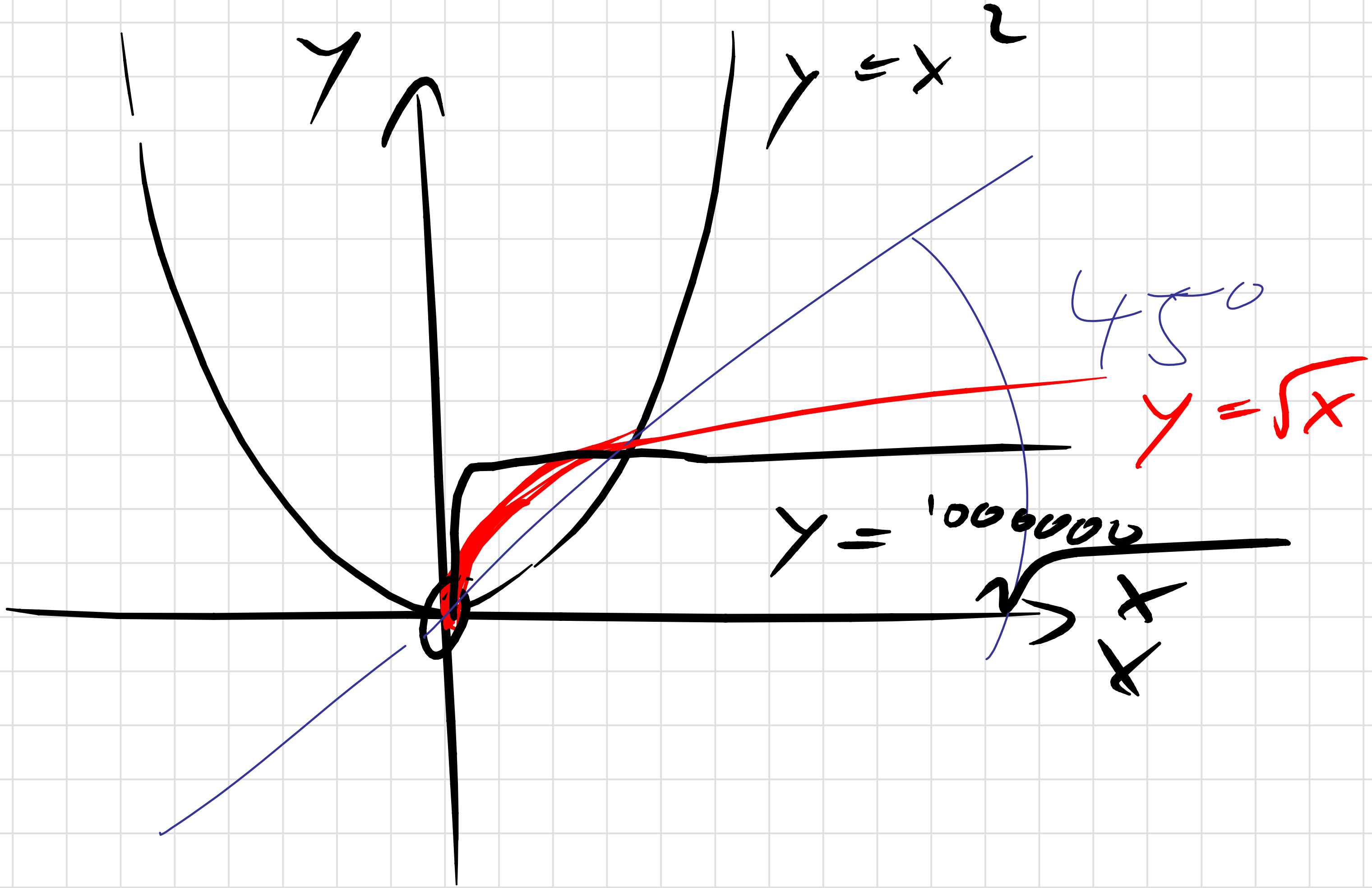
Wurzeln :

$$x^{13} = 5$$

? ! !

$$x = \sqrt[13]{5}$$

$$\sqrt[4]{-3} = ? \iff ?^4 = -3$$



y
$y = x^2$
$45°$
$y = \sqrt{x}$
$y = 1000000\sqrt{x}$
x
x

Würfel hat

Volumen von $4\,m^3 = a^3$

Kantenlänge $a = \sqrt[3]{4\,m^3}$

$$(3^4)^5 = (3\cdot 3\cdot 3\cdot 3)^5$$

$$= 3\cdot 3\cdot 3\cdot 3 \cdot \% \cdot \% \cdot \% \cdot \%$$

$$= 3^{4\cdot 5}$$

$$\boxed{(a^n)^m = a^{n\cdot m}}$$

$$\left(3^{1/4}\right)^4 \overset{!}{=} 3^{\frac{1}{4}\cdot 4} = 3^1 = 3$$

also: $3^{1/4} = \sqrt[4]{3}$

$$3^{71/48} = ?$$

$$\left(3^{\frac{71}{48}}\right)^{48} \overset{!}{=} 3^{\frac{71}{\cancel{48}}\cdot\cancel{48}} = 3^{71}$$

$$\Rightarrow 3^{71/48} = \sqrt[48]{3^{71}}$$

$$3^{71/48} = 3^{\frac{1}{48}\cdot 71} = \left(3^{1/48}\right)^{71}$$

$$= \underline{\left(\sqrt[48]{3}\right)^{71}}$$

$$a^{\frac{n}{m}} = \sqrt[m]{a^n} = \left(\sqrt[m]{a}\right)^n$$

für a, n, m sinnvoll

$$3^{1,4} = 3^{\frac{14}{10}} = \sqrt[10]{3^{14}}$$

$$3^{1,41} = \sqrt[100]{3^{141}}$$

$$3^{1,414} = \sqrt[1000]{3^{1414}}$$

$$\vdots$$

$$3^{\sqrt{2}} = \ldots$$

Potenzfunktionen: $y = x^n$

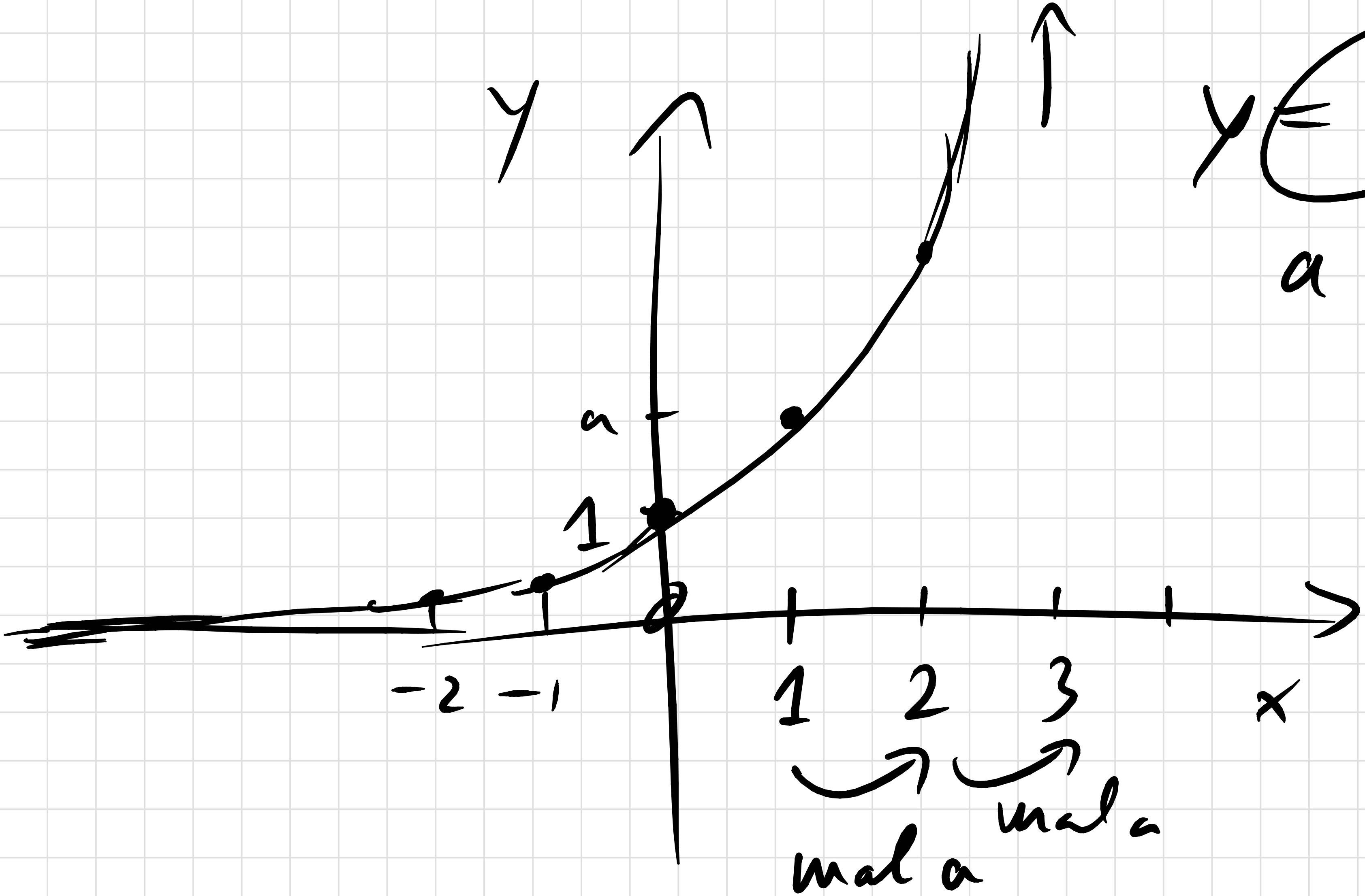
Exponent fest

Basis variabel

Exponentialfunktionen $y = a^x$

Exponent variabel

Basis fest

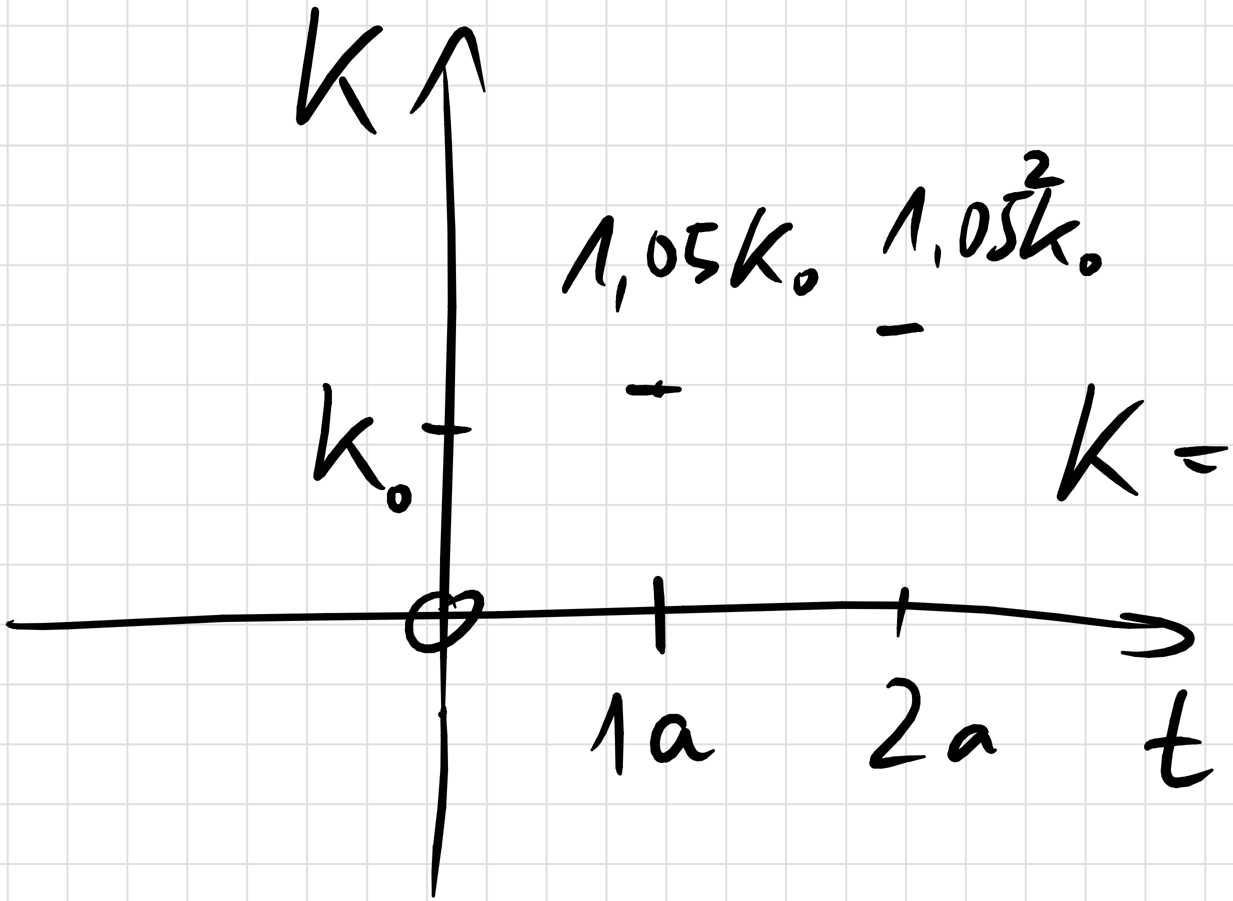


$y = a^x$

$a > 1$

Zinseszins

Zinssatz 5%



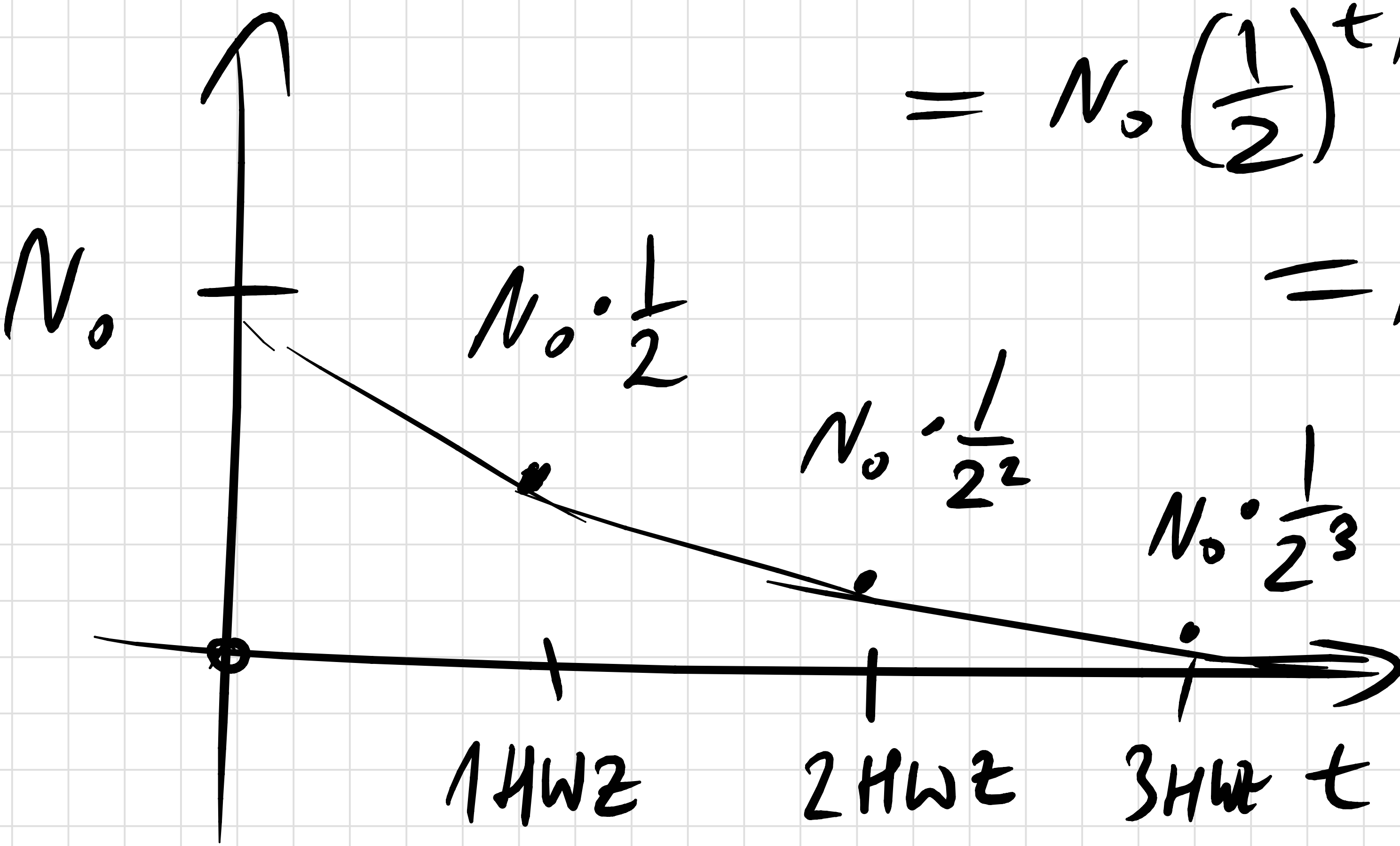
$$K = K_0 \cdot \underline{\underline{1{,}05^{t/1a}}}$$

Zerfall

$$N = N_0 \frac{1}{2^{t/HWZ}}$$

$$= N_0 \left(\frac{1}{2}\right)^{t/HWZ}$$

$$= N_0 2^{-t/HWZ}$$

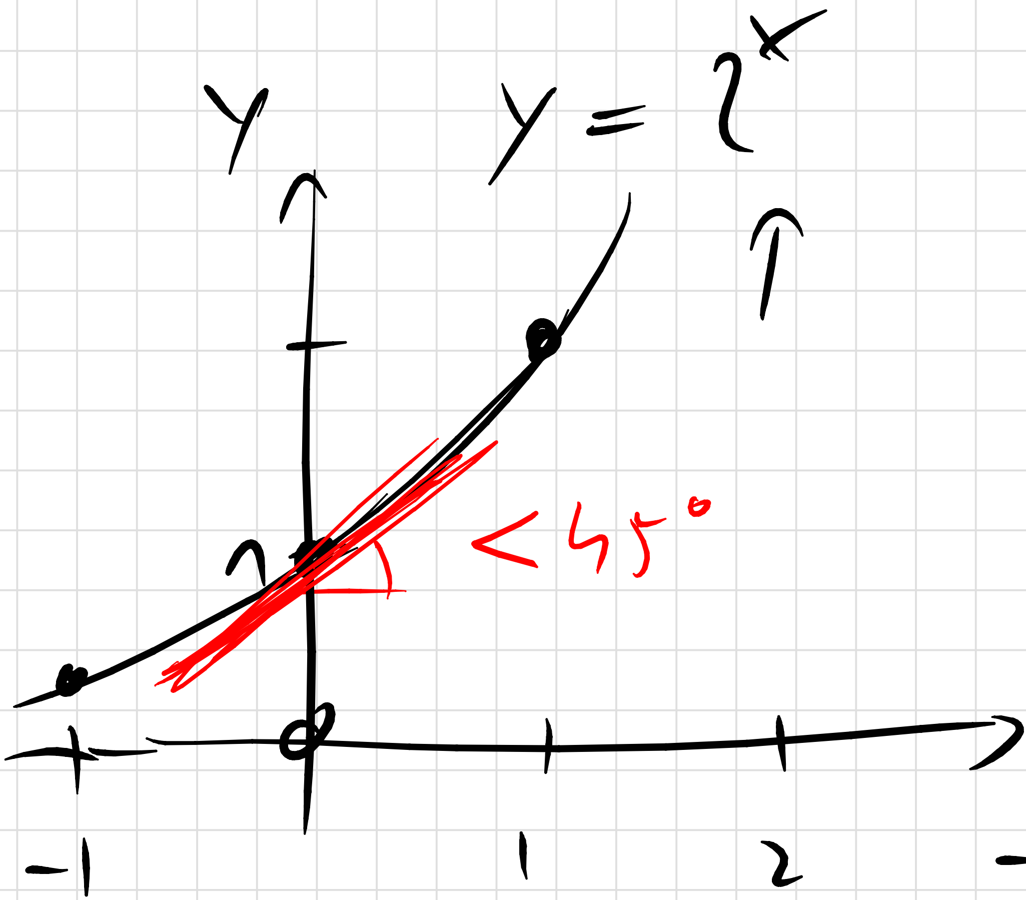


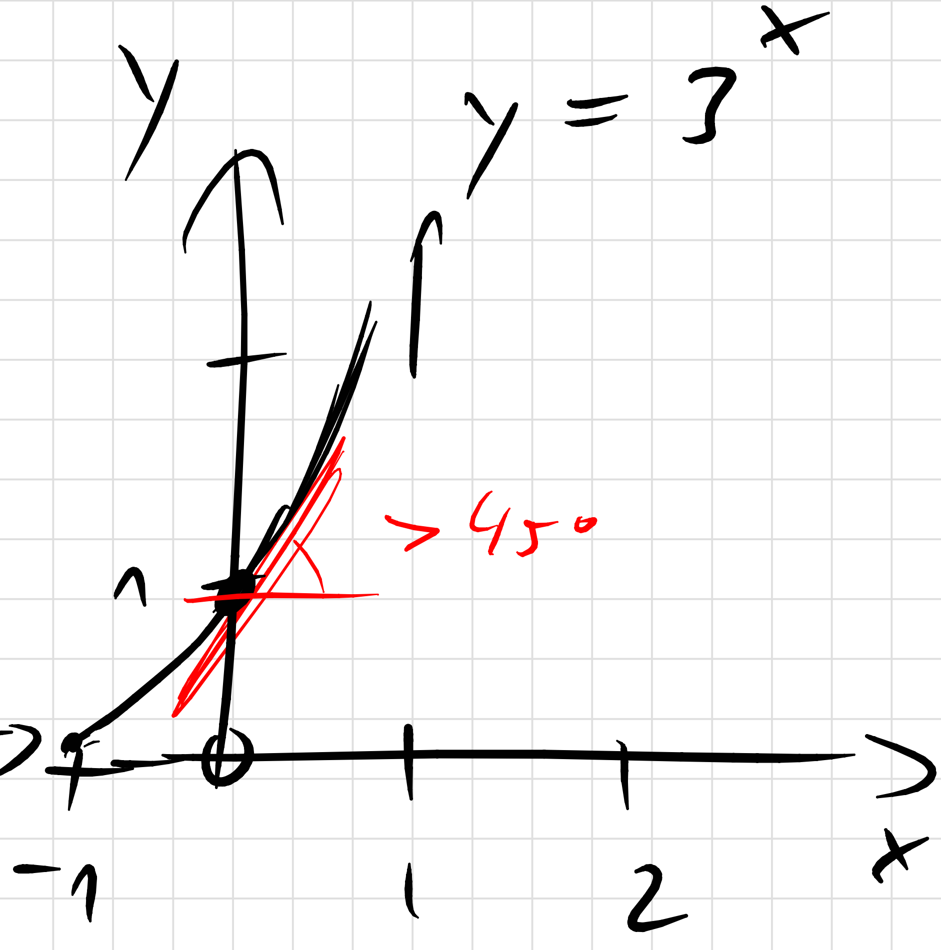
$$\left(\frac{1}{2}\right)^x = \left(2^{-1}\right)^x = 2^{-1\cdot x}$$

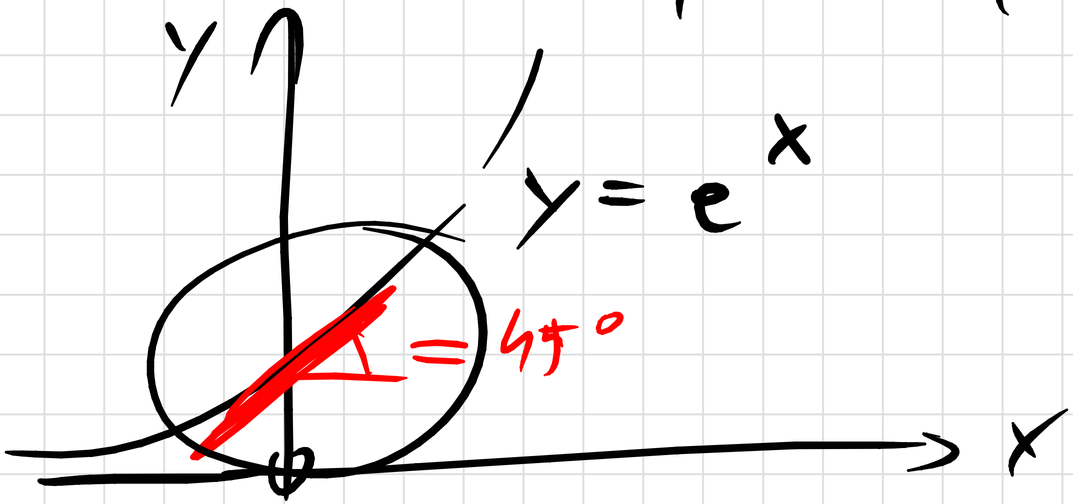
$$= 2^{-x}$$

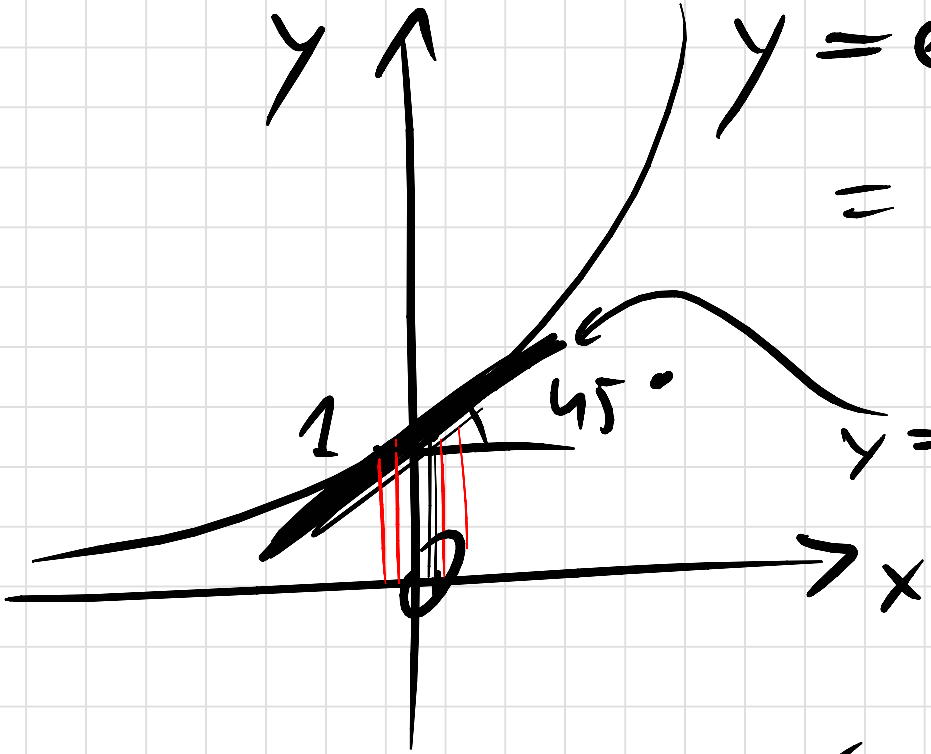
e Eulersche Zahl

$2{,}718\ldots$


y
y = 2^x
1
< 45°
-1
1
2


y
y = 3^x
1
> 45°
-1
1
2
x


y
y = e^x
= 45°
x



$y = e^x$ e-Funktion

$= \exp(x)$

__die__ Exponentialfunktion

$$x \approx 0 \Rightarrow \underline{e^x \approx x + 1}$$

$$e^{0{,}0001} \approx 1{,}000100\ldots$$

$$e = e^{1} = e^{\frac{1000}{1000}}$$

$$= \left(e^{1/1000}\right)^{1000} \approx \left(1 + \frac{1}{1000}\right)^{1000}$$

$$= \left(\frac{1001}{1000}\right)^{1000}$$

$$e := \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^{n} = 2{,}7\ldots$$

$$e^{42,3} = \cancel{e^{423/10} = \sqrt[10]{e^{423}}}$$

$$= e^{\frac{42,3}{1000}\cdot 1000} = \left(e^{\frac{42,3}{1000}}\right)^{1000}$$

$$\approx \left(1+\frac{42,3}{1000}\right)^{1000}$$

Basis

Potenz

Wurzel

Logarithmus

Exponent

$$10^x = 42$$

$$x = \log_{10}(42)$$

$$\log_{10}(\underbrace{1000}_{10^3}) = 3$$

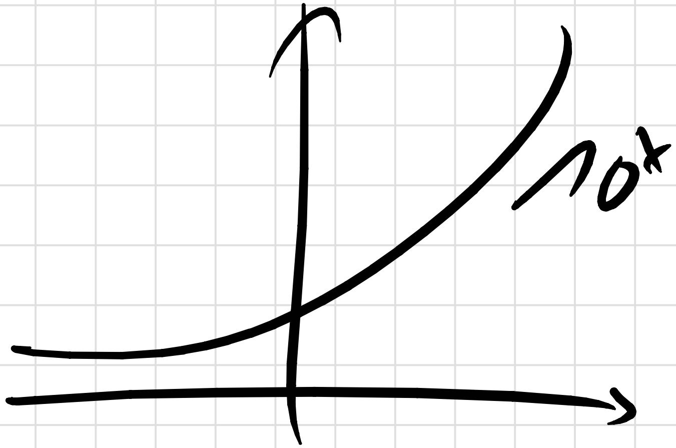
$$\log_{10}(\underbrace{0{,}01}_{10^{-2}}) = -2$$

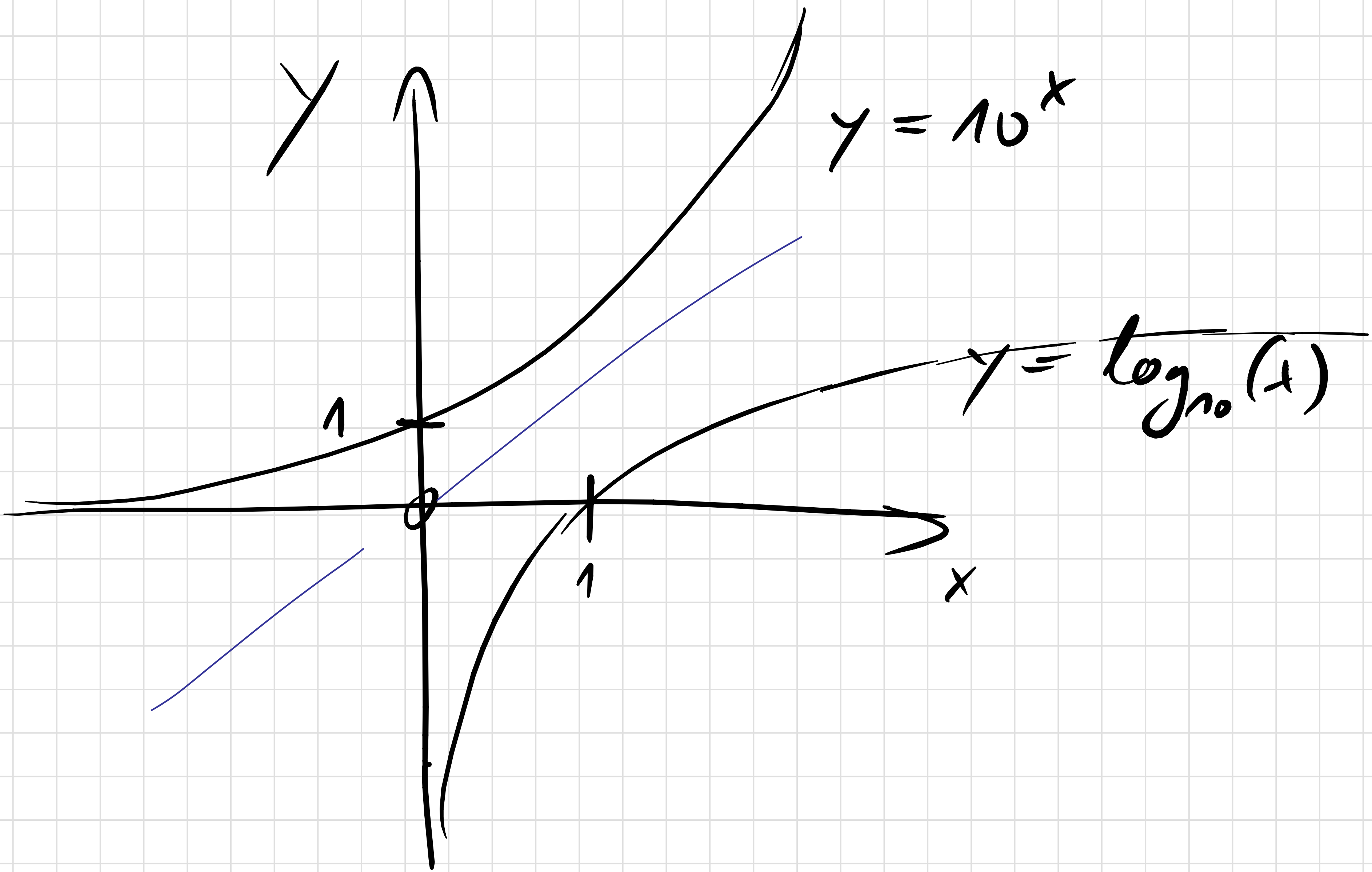
$$10^{-2} = \frac{1}{10^2} = \frac{1}{100}$$

$$\log_{10}(\underbrace{1}_{10^0}) = 0$$

$\cancel{\log_{10}(-100)}$

$\cancel{\log_{10}(0)}$





y
x
1
1
0
y = 10^x
y = log_10(x)

z. B.: 5% Zinsen + Zinseszinz

$100\,€ \longrightarrow 1000\,€$

wieviele Jahre?

$$100\,€ \cdot (1{,}05)^{t/1a} \overset{!}{=} 1000\,€$$

$$\Leftrightarrow (1{,}05)^{t/1a} = \frac{1000\,€}{100\,€} = 10$$

$$\Leftrightarrow \frac{t}{1a} = \log_{1{,}05} 10$$

$$\Leftrightarrow t = 1a \cdot \log_{1{,}05} 10$$

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64 mögliche Zeichen,

Wie viele Bits nötig, um die darzustellen?

$$2^x = 64 \Leftrightarrow x = \log_2 64$$

$$= 6$$

$$\log_{10}(3\cdot 4) = \otimes$$

$$\Updownarrow$$

$$10^{x} = 3\cdot 4 = \underbrace{10^{\log_{10} 3}}_{3} \cdot \underbrace{10^{\log_{10} 4}}_{4}$$

$$= 10^{(\log_{10} 3) + (\log_{10} 4)}$$

$$\Rightarrow \otimes = \log_{10} 3 + \log_{10} 4$$

$$\log_b(x \cdot y) = \log_b(x) + \log_b(y)$$

$$\log_b\left(\frac{x}{y}\right) = \log_b(x) - \log_b(y)$$

$$\log_b(x^y) = y \cdot \log_b(x)$$

$$\log_b(\sqrt[y]{x}) = \frac{\log_b(x)}{y}$$

im log		außen
$\cdot$	$\longrightarrow$	$+$
$:$	$\longrightarrow$	$-$
$\square^{\square}$	$\longrightarrow$	$\cdot$
$\sqrt[\square]{\square}$	$\longrightarrow$	$:$
$+$	$\longrightarrow$	⚡
$-$	$\longrightarrow$	⚡

$$\log_{10}\left(\cancel{x}^3 + 4\cancel{x}^2 + 23\right)$$

$$10^x$$

$$= \log_{10}\left(\underbrace{(10^x)^3 + 4(10^x)^2 + 23}_{10^{3x} + 4\cdot 10^{2x} + 23}\right)$$

$$\log_{10}\left(10^{3x}\right) = 3x$$

$$\lg = \log_{10}$$

$$\ln = \log_e$$

$$\text{lb} = \text{ld} = \log_2$$

$$\log_3(4) = x$$

$$\Longleftrightarrow \quad 3^x \stackrel{!}{=} 4 = 10^{\boxed{\log_{10} 4}}$$

$$\left(10^{\log_{10} 3}\right)^x = 10^{\boxed{x \log_{10} 3}}$$

Also: $x \log_{10} 3 = \log_{10} 4$

$$\Longrightarrow \quad x = \frac{\log_{10} 4}{\log_{10} 3}$$

$\log_3 4$

Allgemein: $\boxed{\log_b a = \frac{\log_{10} a}{\log_{10} b}}$

$$\log_{1,05} 10 = \frac{\log_{10} 10}{\log_{10} 1,05} = 1$$

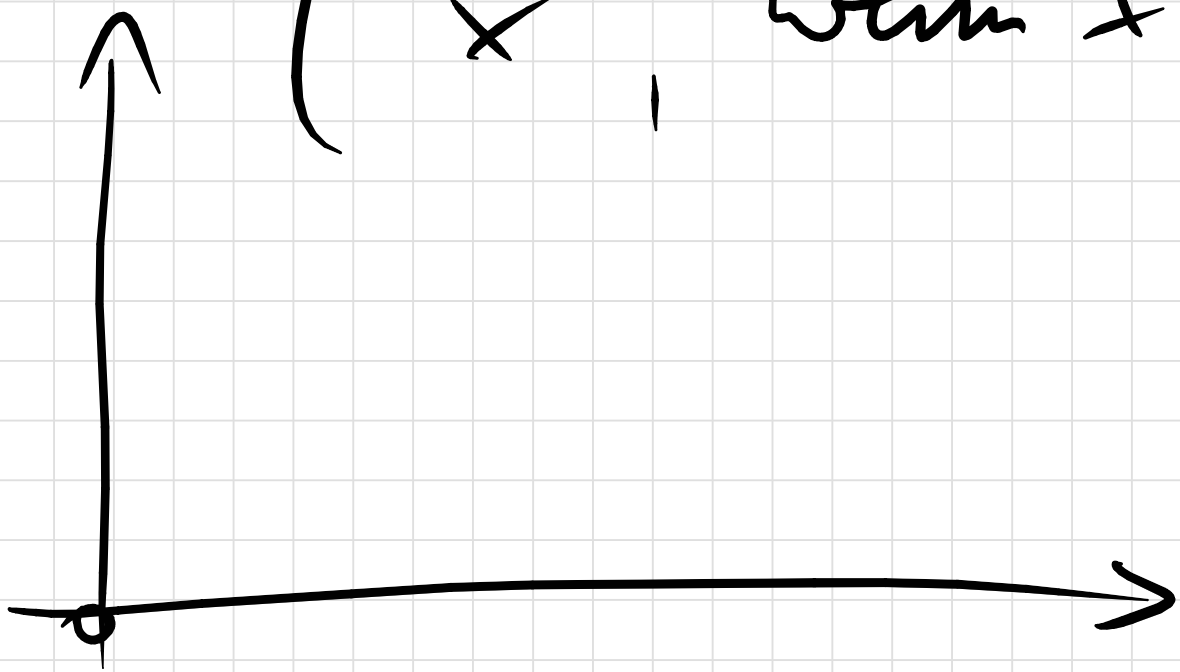
Absoluter Betrag $|\ldots|$

$$|42| = 42$$

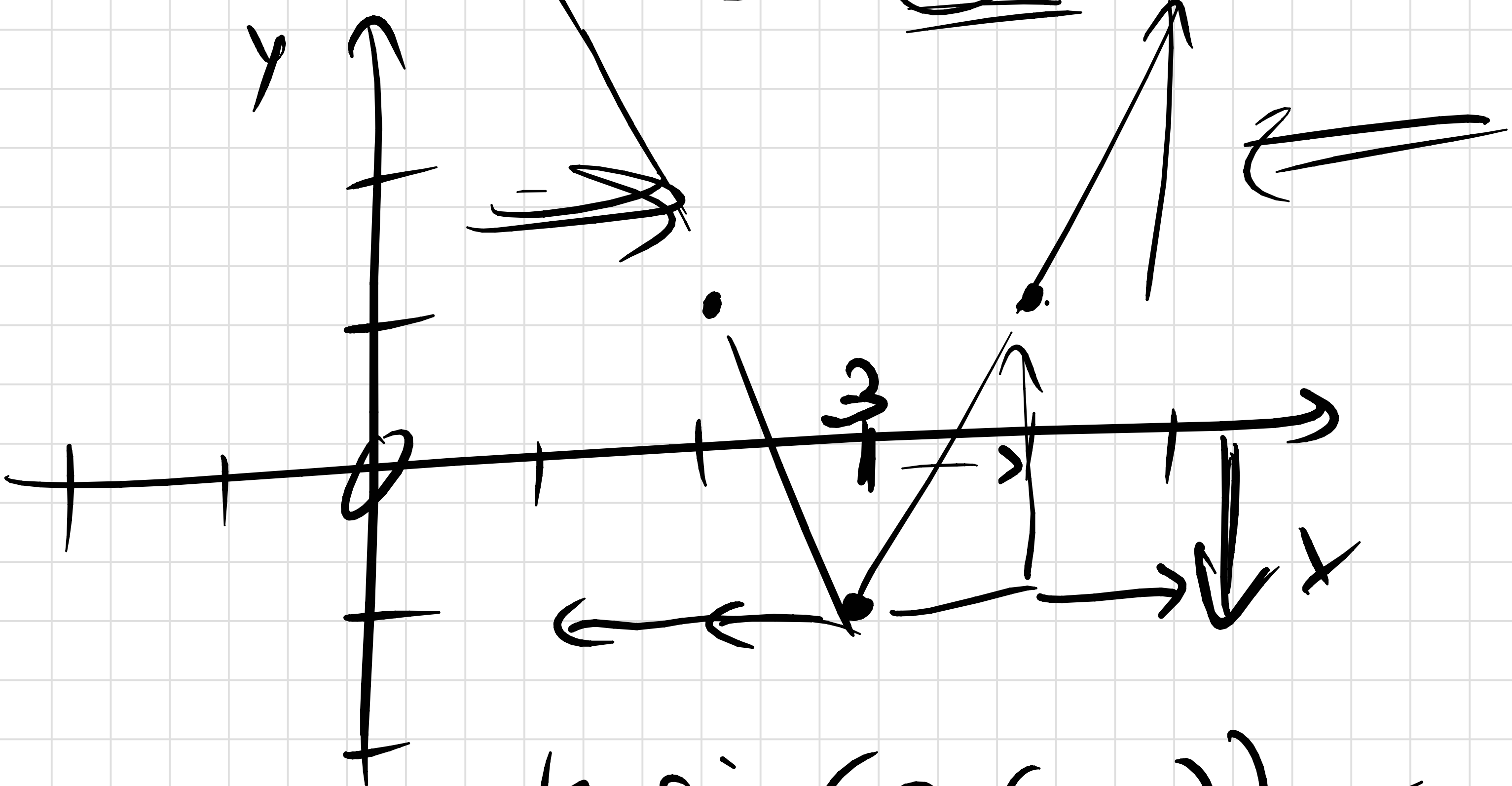
$$|-13| = 13$$

$$|x| := \begin{cases} x, & \text{wenn } x \geq 0 \\ -x, & \text{wenn } x < 0 \end{cases}$$

$$Y = \begin{cases} 5, & \text{wenn } x < 2 \\ 1, & \text{wenn } x \geq 2 \wedge x < 3 \\ x, & \text{wenn } x \geq 3 \end{cases}$$



$y = 2|x-3|-1$



$4\sin(2(x-3))+5$