Quadratische Gleichungen

$$3x^2 - 5x + 1 = 0 \quad \| :3$$

$$\Leftrightarrow \quad x^2 \overbrace{- \frac{5}{3}}^{p} x + \overbrace{\frac{1}{3}}^{q} = 0$$

Normalform

$$\Leftrightarrow \quad \boxed{x^2 - \frac{5}{3}x + \left(\frac{5}{6}\right)^2} - \left(\frac{5}{6}\right)^2 + \frac{1}{3} = 0$$

quadratische Ergänzung

$$x^2 + 2x\cdot\left(-\frac{5}{6}\right) + \left(\frac{5}{6}\right)^2$$

$$\Longleftrightarrow \left(x - \frac{5}{6}\right)^2 - \left(\frac{5}{6}\right)^2 + \frac{1}{3} = 0$$

$$a \quad + \quad b$$

$$\Longleftrightarrow \left(x - \frac{5}{6}\right)^2 = \left(\frac{5}{6}\right)^2 - \frac{1}{3}$$

glücklicherweise ≥ 0

$$\Longleftrightarrow x - \frac{5}{6} = \pm\sqrt{\left(\frac{5}{6}\right)^2 - \frac{1}{3}}$$

$$\Longleftrightarrow x = \frac{5}{6} \pm \sqrt{\left(\frac{5}{6}\right)^2 - \frac{1}{3}}$$

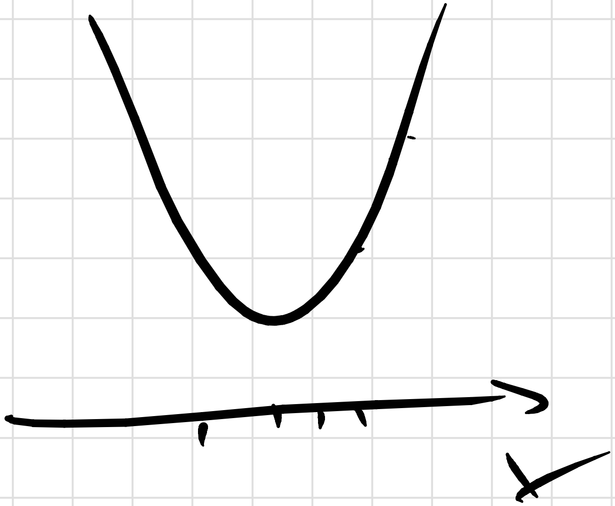
Allgemein: $x = -\frac{p}{2} \pm \sqrt{\underbrace{\frac{p^2}{4} - q}}$

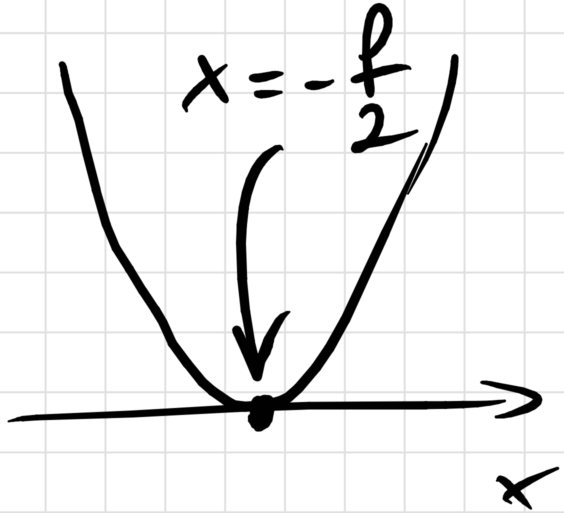
wenn

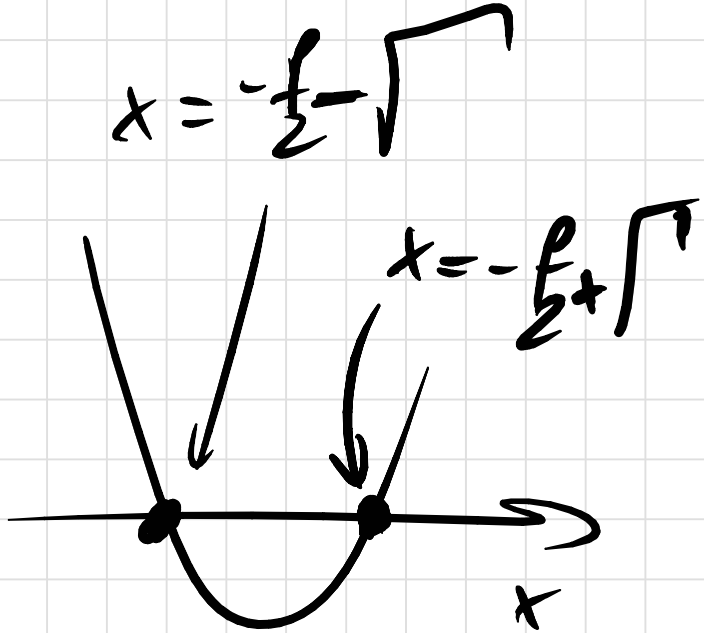
≥ 0,

sonst keine Lösung

$$y = x^2 + px + q$$







$\frac{p^2}{4} - q < 0$

$\frac{p^2}{4} - q = 0$

$\frac{p^2}{4} - q > 0$

$$y = 3x^2 - 5x + 1$$

$$= 3 \cdot \underbrace{\left(x - \left(\tfrac{5}{6} - \sqrt{\ldots}\right)\right)} \cdot \underbrace{\left(x - \left(\tfrac{5}{6} + \sqrt{\ldots}\right)\right)}$$

1. Nullstelle

2. Nullstelle

„Linearfaktor"

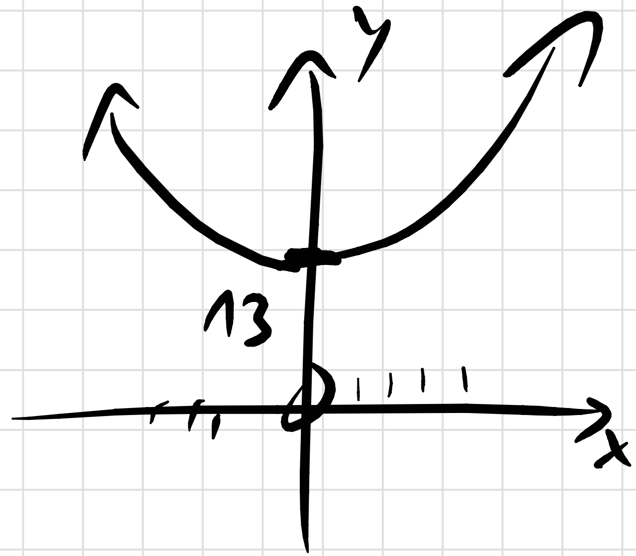
$$y = \underbrace{4x^5 - 3x^4 + 7x^3 - 9x^2 + 2x + 8}_{\text{Polynom 5. Grades}}$$

ganzrationale Funktion

$= 4 \cdot (x - \text{Nullstelle}) \cdot (x - \text{Nullstelle})$

$\cdot (\ldots) \cdot$ Polynom ohne Nullstellen

Beispiel : $y = x^6 + x^2 + 13$



Faktorisierung

- natürliche Zahlen

$$\frac{210}{3}$$

$$420 = 2 \cdot 210 = 2 \cdot 3 \cdot 70 = 2 \cdot 3 \cdot 7 \cdot 10$$
$$= 2 \cdot 3 \cdot 7 \cdot 2 \cdot 5 = 2^2 \cdot 3 \cdot 5 \cdot 7$$

- Polynome

$$x \dots x - \dots = (x - \dots)(x^4 \dots)$$
$$= (x - m)(x - m)(x^3 \dots) = \dots$$

$y = x^3 - 2x^2 - 5x + 6$

hat die Nullstelle $x = 3$,

denn:

$3^3 - 2 \cdot 3^2 - 5 \cdot 3 + 6$

$= 27 - 18 - 15 + 6 = 0$

Also: $x^3 - 2x^2 - 5x + 6 = (x-3) \cdot (x^2 \ldots)$

Polynomdivision muss aufgehen:

$$(x^3 - 2x^2 - 5x + 6) : (x-3) = x^2 + x - 2$$

$$\begin{array}{l} -(x^3 - 3x^2) \\ \hline \quad x^2 - 5x \\ \quad -(x^2 - 3x) \\ \hline \qquad -2x + 6 \\ \qquad -(-2x + 6) \\ \hline \qquad\qquad 0 \end{array}$$

Rest 0

$210 : 7 = 30$ Rest 0

$\Rightarrow 210 = 7 \cdot 30$

Also: $x^3 - 2x^2 - 5x + 6 = (x-3)(x^2 + x - 2)$

$$591 : 40 = 14 \text{ Rest } 31$$

$$\begin{array}{r} 591 \\ -40 \\ \hline 191 \\ 160 \\ \hline 31 \end{array}$$

Also $\dfrac{591}{40} = 14\dfrac{31}{40}$.

$$x^3 - 2x^2 - 5x + 6 = (x-3)\underbrace{(x^2+x-2)}_{\text{Nullstellen ?}}$$

$$\text{NR: } x^2 + x - 2 = 0$$

$$\Leftrightarrow x = -\frac{1}{2} \pm \sqrt{\frac{1}{4}+2}$$

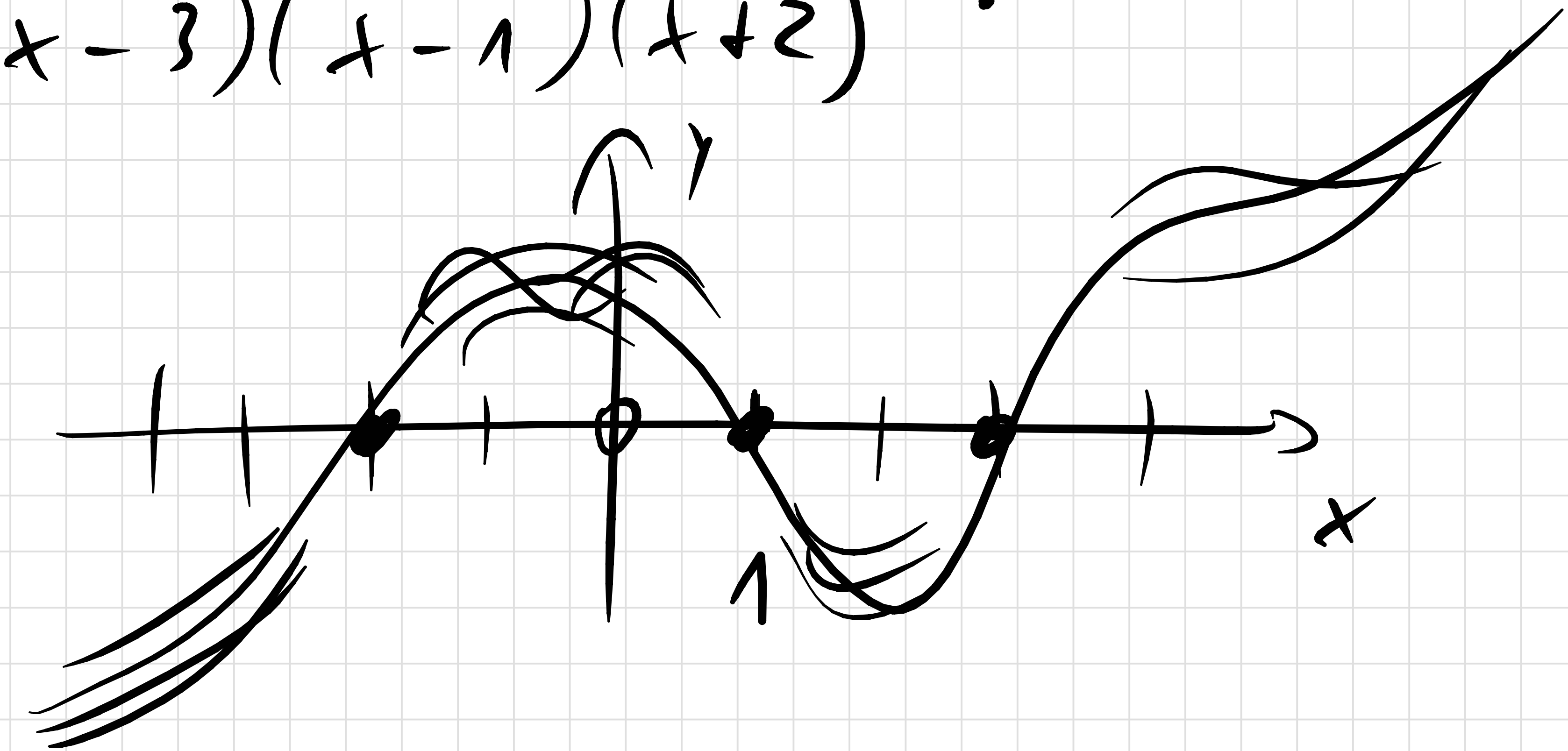
$$= -\frac{1}{2} \pm \sqrt{\frac{9}{4}} = \frac{-1 \pm 3}{2}$$

$$\Leftrightarrow x = 1 \lor x = -2$$

$$x^3 - 2x^2 - 5x + 6 = (x-3)\underbrace{(x^2+x-2)}_{(x-1)(x+2)}$$

?!

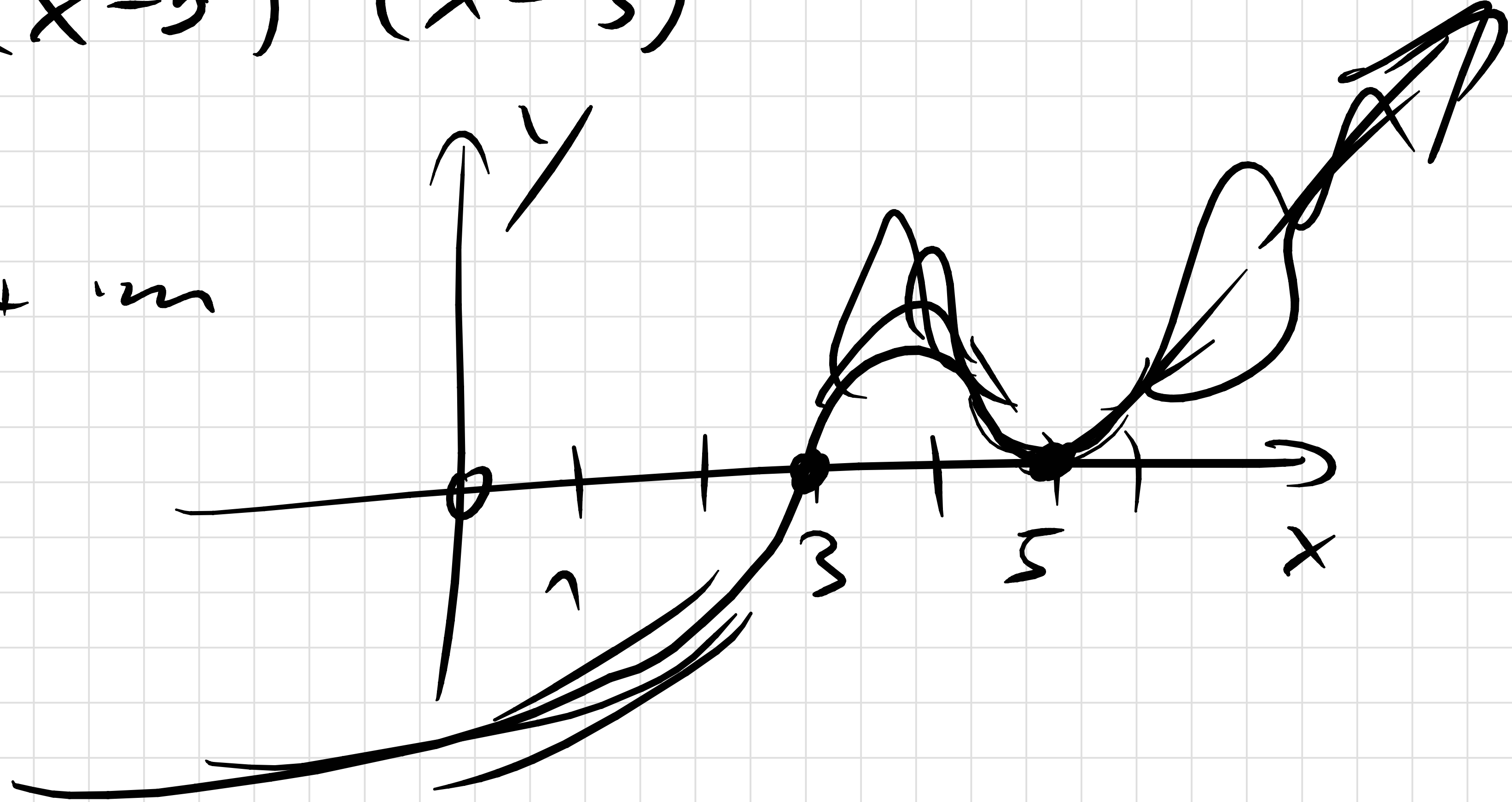
$$= (x-3)(x-1)(x+2)$$



Es hätte auch passieren können:

$$y = 4(x-5)^2(x-3)$$

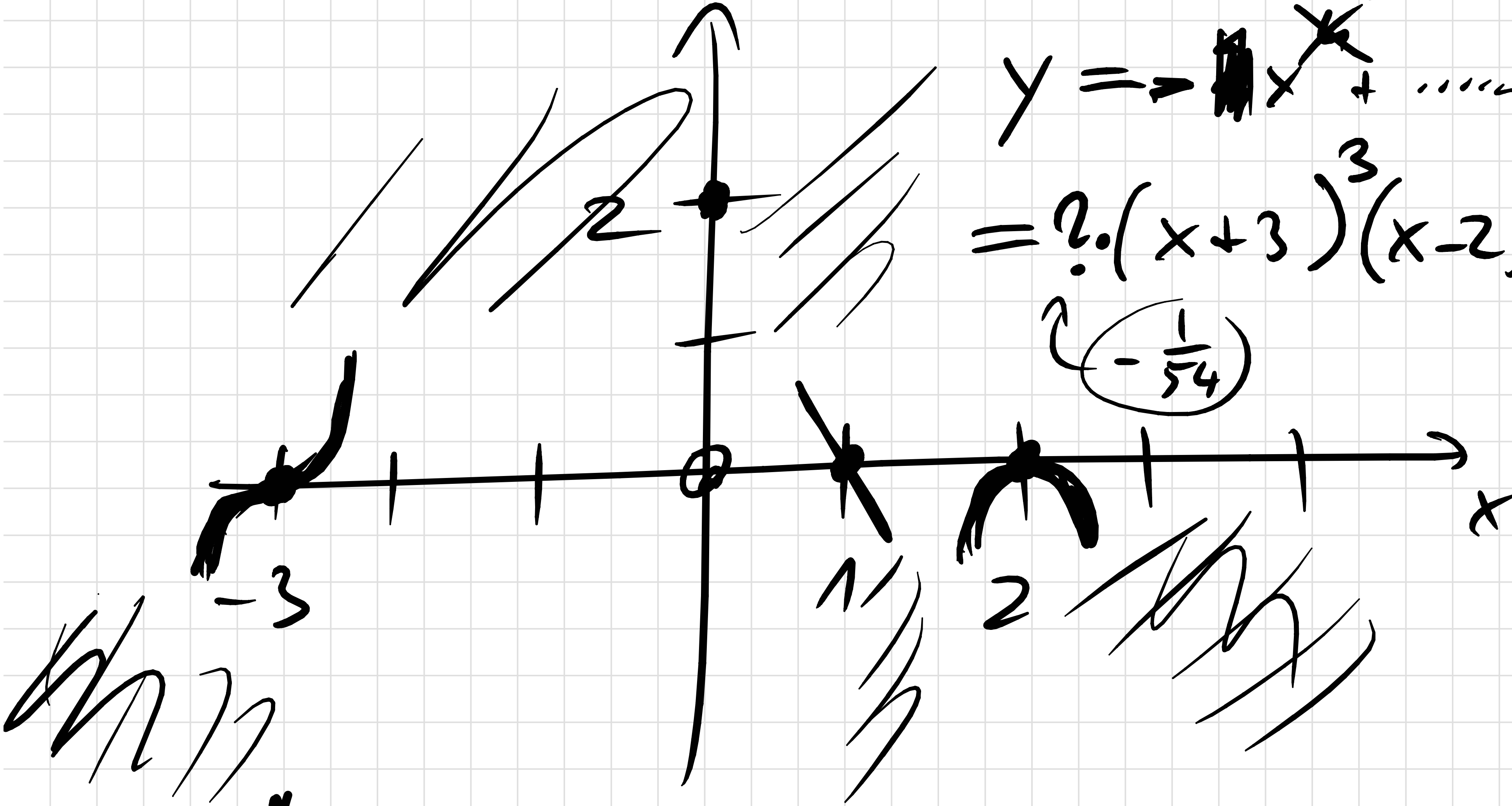
$4x^3 + \dots$



$$y = -\cancel{}x^{6} + \dots + 2$$

$$= ?\cdot(x+3)^3(x-2)^2(x-1)$$

$$-\frac{1}{54}$$



$$2 \overset{!}{=} ?\,\underbrace{(0+3)^3}_{27}\underbrace{(0-2)^2}_{4}\underbrace{(0-1)}_{-1} = -108\cdot ?$$

$$\Rightarrow \quad ? = -\frac{2}{108} = -\frac{1}{54}.$$

$$\frac{7}{3} = 2\overset{\swarrow \text{Rest}}{\frac{1}{3}} = 2 + \frac{1}{3}$$

$$7 : 3 = 2 \text{ Rest } 1$$

Genauso bei Polynomen:

$$\frac{x^3 - 2x^2 - 5x^{\cancel{2}} + 6}{x^2 + 2x - 2} = x\,m + \frac{\quad}{x^2 + 2x - 2}$$

$$(x^3-2x^2-5x+6):(x^2+2x-2)=x-4$$

$$-(x^3+2x^2-2x)$$

$$-4x^2-3x+6$$

$$-(-4x^2-8x+8)$$

$$5x-2$$

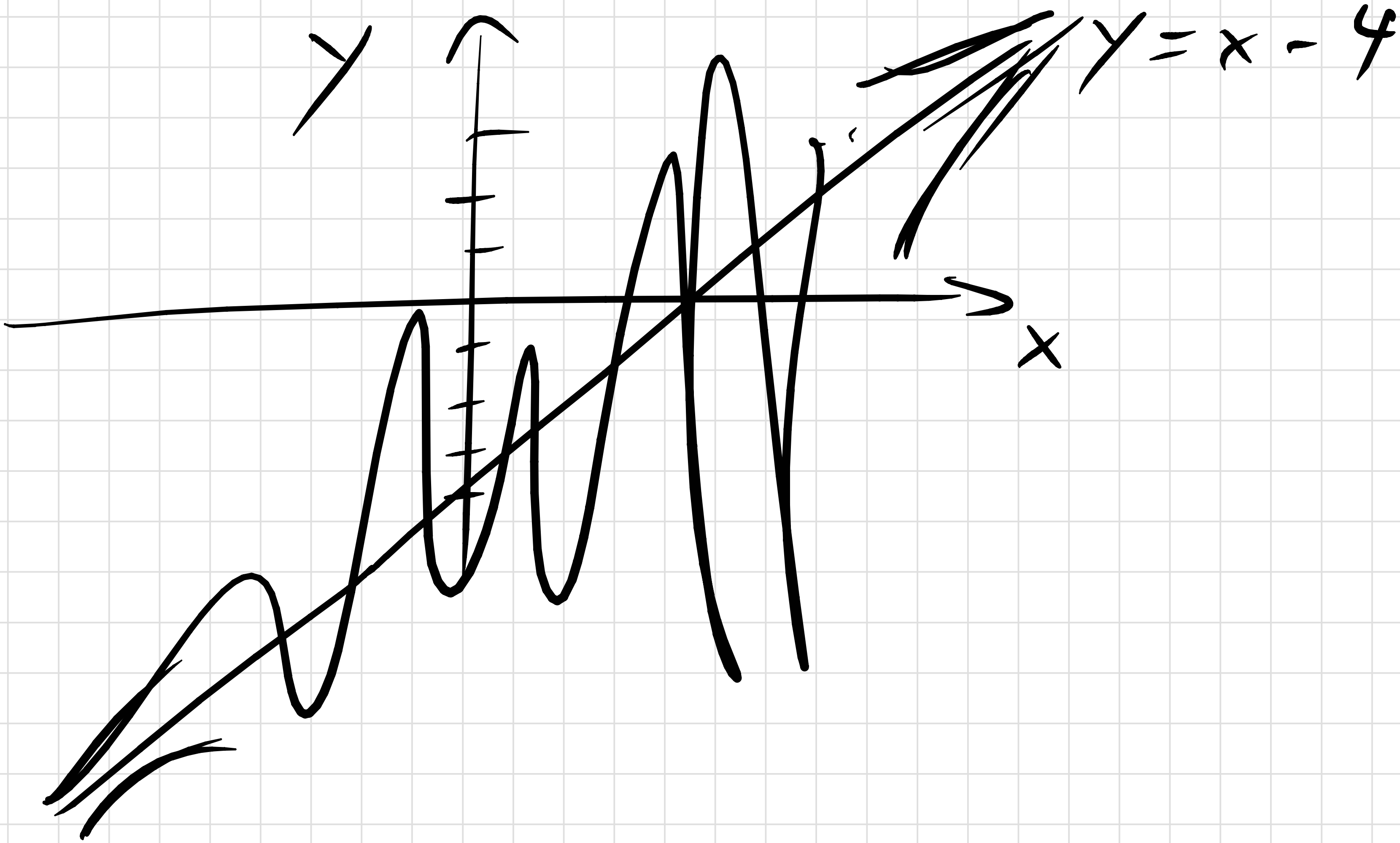
Rest $5x-2$

$7:3=2\ R1$

$\Rightarrow 7=2\cdot 3+1$

$$\frac{x^3-2x^2-5x+6}{x^2+2x-2}=x-4+\frac{5x-2}{x^2+2x-2}$$

$$7/3 = 2+1/3$$


y
x
y = x - 4

biquadratische Gleichung

$$x^4 + px^2 + q = 0$$

$u = x^2$ Substitution

$$\Rightarrow u^2 + pu + q = 0$$

Löse nach $\underline{u_{1,2}}$ auf! Wenn $u_{1,2} \geq 0$:
Dann $x = \pm\sqrt{u_{1,2}}$. Max. vier Lösungen.

$$\frac{x^8 - 3x^6}{2x^2+5} \overset{!}{=} 7$$

substituiere $x^2 =: u$

$$\frac{(3x+8)^8 - 3(3x+8)^6}{2(3x+8)^2+5} \overset{!}{=} 7$$

substituiere $(3x+8)^2 =: u$.

Ungleichungen

- $3x + 4 \overset{!}{<} 7 \iff 3x < 3$

$\iff x < 1 \iff x \in (-\infty, 1)$

$\not\Leftrightarrow \cancel{x \in \{u \in \mathbb{R} :} \ \underline{u < 1}\}$

- $-3x + 4 \overset{!}{<} 7 \iff -3x < 3$

$\iff x > -1$

z.B. $-42 < 3 \quad // :(-3)$

$14 > -1$

- $|3x+4| < 7$

$3x+4 \geq 0 \wedge 3x+4 < 7$

$\Longleftrightarrow$

$\vee \quad 3x+4 < 0 \wedge -3x-4 < 7$

NR

$-3x < 11$

$x > -\frac{11}{3}$

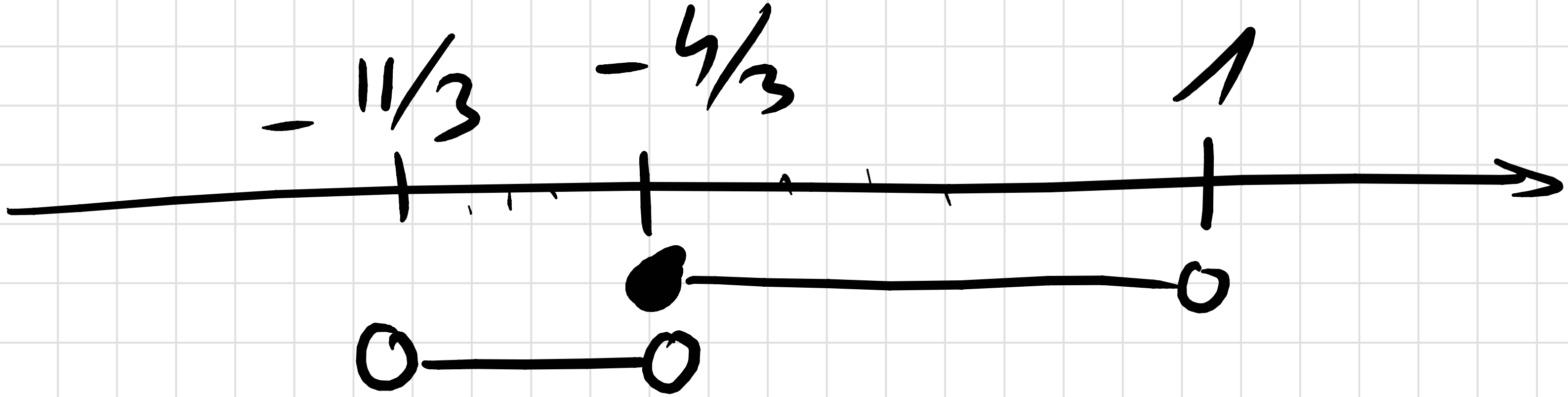
$\Longleftrightarrow \quad x \geq -\frac{4}{3} \wedge x < 1$

$$\vee \quad x < -4/3 \wedge x > -\frac{11}{3}$$

$$\iff \quad x \in [-4/3, 1)$$

$$\vee \quad x \in \left(-\frac{11}{3}, -\frac{4}{3}\right)$$

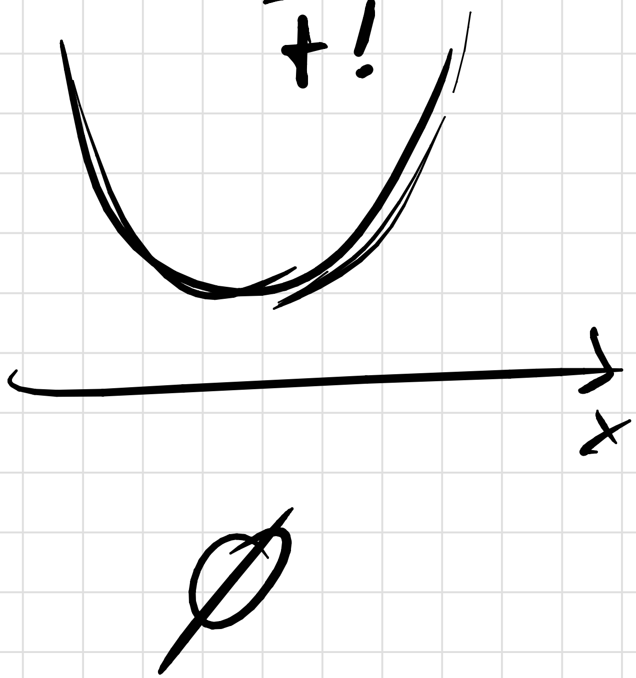
$$\iff \quad x \in (-11/3, 1)$$

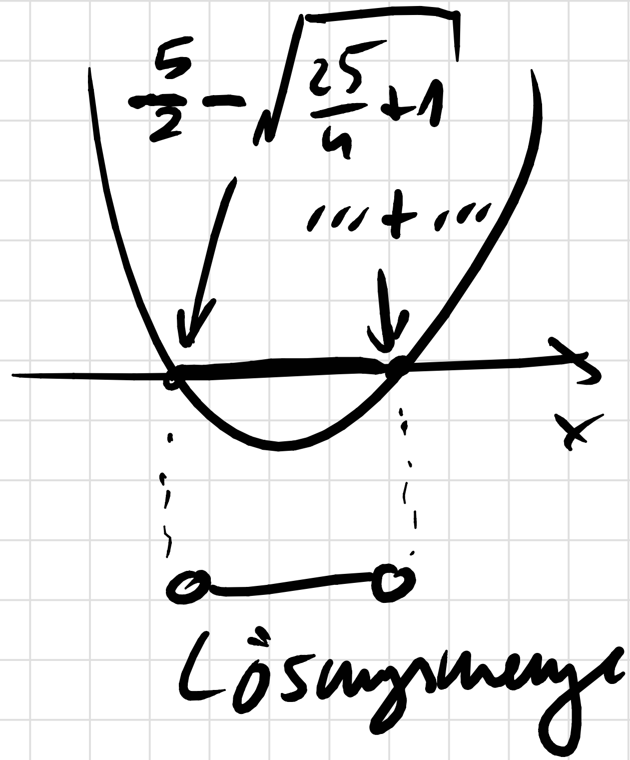


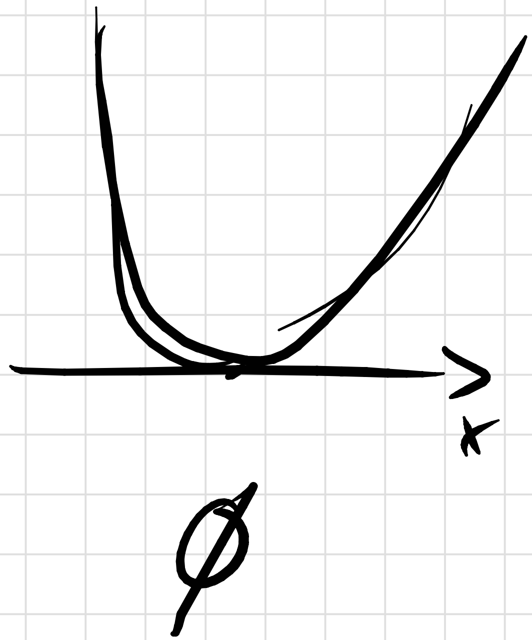
- $x^2 - 5x + 6 < 7$

$\Longleftrightarrow x^2 - 5x - 1 < 0$

+!







- $$\frac{x+4}{x-3} < 7 \quad \text{mit} \quad x \overset{!}{\neq} 3$$

$$\Longleftrightarrow \; x-3>0 \wedge x+4 < 7\cdot(x-3)$$

$$\vee \; x-3<0 \wedge x+4 > 7\cdot(x-3)$$

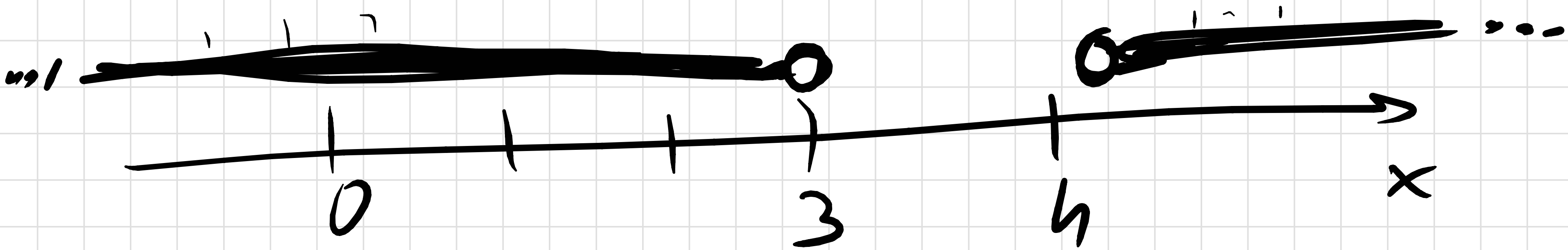
$$\Longleftrightarrow \; x>3 \wedge \underline{x+4 < 7x-21}$$

$$\vee \; x<3 \wedge x+4 > 7x-21$$

$$\Longleftrightarrow \quad x > 3 \wedge 25 < 6x$$
$$\vee \quad x < 3 \wedge 25 > 6x$$

$$\Longleftrightarrow \quad x > 3 \wedge x > 4\tfrac{1}{6}$$
$$\vee \quad x < 3 \wedge x < 4\tfrac{1}{6}$$

$$\Longleftrightarrow \quad x > 4\tfrac{1}{6} \vee x < 3$$



$$\Longleftrightarrow \quad x \in (-\infty, 3) \cup (4\tfrac{1}{6}, \infty)$$

Vereinigung $\cup$ Schnitt $\cap$

$A \cup B$: in A $\check{\text{oder}}$ in B

$A \cap B$: in A $\hat{\text{und}}$ in B

Gleichungssystem:

$$\begin{cases} x^2 + \sin(y) = 3 \\ \sqrt{x e^x} = 4^y \end{cases}$$

Lineares Gleichungssystem:

$$\begin{cases} 3x + 4y = 5 \\ 7x - 2y = 3 \end{cases}$$

$$\Longleftrightarrow \begin{cases} x = (5 - 4y)/3 \\ 7x - 2y = 3 \end{cases}$$

$$\Longleftrightarrow \begin{cases} x = (5-4y)/3 \\ 7\,\dfrac{5-4y}{3} - 2y = 3 \end{cases}$$

$$\Longleftrightarrow \begin{cases} x = (5-4y)/3 \\ -\dfrac{28}{3}y - 2y + \cancel{\dfrac{35}{3}} = 3 - \dfrac{35}{3} \end{cases}$$

$$\Longleftrightarrow \begin{cases} x = \dots \\ y = \dots \end{cases}$$

$$\begin{cases} 3x + 4y = 5 \\ 3x + 4y = 6 \end{cases}$$

keine Lösung

$$\begin{cases} 3x + 4y = 5 \\ 9x + 12y = 1000 \end{cases}$$

"

$$\begin{cases} 3x + 4y = 5 \\ 9x + 12y = 15 \end{cases}$$

∞ viele Lösungen

$$y = \frac{5 - 3x}{4}$$

frei

$$\begin{cases} 0x + 0y = 0 \\ 0x + 0y = 0 \end{cases}$$

x, y beliebig

nichtlineare nichtalgebraische Gleichung

$$10^{3\sqrt{x}+2} \overset{!}{=} 5 = 10^{\log_{10}(5)}$$

$\log_{10}(5) < 1$

$$\Leftrightarrow \quad 3\sqrt{x}+2 = \log_{10}(5)$$

$$\Leftrightarrow \quad \sqrt{x} = \Big(\underbrace{\log_{10}(5)}_{<1} - 2\Big)/3 < 0 \quad ↯$$

keine Lösung

$$x^2 = 4$$

$$\Leftrightarrow x = \pm\sqrt{4} \Leftrightarrow x = 2$$
$$\vee\, x = -2$$

$$\sqrt{x} = -2 \quad \Big/\Big/ \;\cdot^2$$

$$x = 4$$

$$\underbrace{10^{\sqrt[3]{x}+2}}_{f(x)} \overset{!}{=} \underbrace{\frac{1}{x}}_{g(x)}$$